

# Portfolio Allocation over Life-Cycle with Multiple Late-in-Life Saving Motives

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## Abstract

Older households face health-related risks, including risk of being in need of long-term care and mortality risk. How these risks affect financial portfolio choice of households depends on household preferences for long-term care and bequest. Using linked survey-administrative data on clients of a mutual fund company, this paper finds that the desire to have enough resources for long-term care and bequests are overall strong but also heterogeneous across households. The estimated relationship between actual stock share of households and the strength of these preferences is qualitatively similar but quantitatively weaker compared to the predictions from the life-cycle model with the estimated preference heterogeneity. Based on the predictions from the model, this paper discusses what financial instruments would better meet the needs of households.

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# 1 Introduction

Older households face multiple risks in retirement. Most importantly, they face health-related risks, including considerable expenditure risk associated with long-term care (LTC) and mortality risk. Given the high cost of LTC, the risk of being in need of LTC effectively increases households' risk aversion, limiting their ability to take additional risks in the financial market for a higher expected return. Mortality risk adds another layer of uncertainty that may further reduce room for risky assets in households' financial portfolio. In household portfolio choice literature, relatively little attention has been given to the implications of these health-related background risks on portfolio choice, in particular on the choice of the share of risky assets. Instead, most research on household portfolio choice has focused on the effects of labor income uncertainty (see Benzoni, Collin-Dufresne, and Goldstein, 2007; Bodie, Merton, and Samuelson, 1992; Cocco, Gomes, and Maenhout, 2005; and Viceira, 2001, among others), which is not a major source of background risk for households that are near or in retirement. This study addresses this gap in the literature by examining how these health-related background risks affect portfolio allocation over the life-cycle.

Health-related risks have complex effects on decisionmaking of households because they likely affect preferences directly. Hence, how they affect asset accumulation, portfolio choices, and spending will depend on preferences about related expenditures. For example, for households who have a preference for high-quality, expensive service when they need LTC, the same probability of being in need of LTC implies effectively a much larger expenditure risk. Similarly, two households with equal mortality risk may choose different portfolios depending on the strength of the bequest motive. Moreover, the paper will show that there are complicated and subtle interactions of preferences over LTC and bequests with health-related risks.

This paper uses distinctive modeling approach and measurement infrastructure to study the financial decisionmaking of households facing these late-in-life risks. The approach uses novel survey instruments, introduced in Ameriks, Briggs, Caplin, Shapiro, and Tonetti (2015, 2016), to identify preferences relevant to late-in-life portfolio choices. It uses survey responses to quantify the distribution of these preferences in the population and then to relate these preferences to choices and outcomes in a new dataset—the Vanguard Research Initiative (VRI)—that combines survey and administrative account information on a large population of older Americans who have sufficient financial assets to make these portfolio choices highly relevant.

Specifically, I first estimate households' preferences for expenditures in the LTC state and bequests

using responses to hypothetical survey questions. Estimated utility functions for LTC expenditures and bequests not only show the strength of the precautionary saving motive for LTC and the bequest motive over the life-cycle, but also govern households' exposure to health-related expenditure risks for a given amount of resources. I then investigate the empirical relationship between the estimated strength of these two saving motives and actual stock share of households to see whether households' actual portfolio choice responds to health-related background risks. I also study how the optimal stock share should respond to the strength of these saving motives using a life-cycle portfolio choice model with the estimated preference heterogeneity. Lastly, I contrast the findings from the empirical and theoretical analyses to derive implications for a better design of financial advice and financial products.

The responses to the hypothetical survey questions suggest that the preferences for LTC expenditures and bequests are both strong but also heterogeneous among households. For many households, the estimated preference parameters suggest that their life-cycle saving is mainly driven by a precautionary motive associated with LTC or a bequest motive. On the other hand, a non-trivial fraction of households appear to put much larger weight on their consumption in the state of good health than LTC expenditures or bequests.

My analysis using the life-cycle model with the estimated preference heterogeneity suggests that both a stronger preference for LTC expenditures and a stronger bequest motive imply lower optimal stock share. The more households care about expenditures in the LTC state, the more painful a combination of a negative stock return shock and an LTC shock is. The impact of the LTC preference is limited for households with fewer resources, because for them publicly-funded nursing home functions as a partial insurance against negative stock return shocks. The mechanism behind the effect of the bequest motive is subtler. On one hand, most households consider bequest as a luxury good, which effectively makes households who put more weight on bequests than consumption less risk averse. On the other hand, the existence of retirement income and LTC risk under the presence of mortality risk makes households with a stronger bequest motive more reluctant to take risks in the financial market, compared to households who mainly care about own consumption. I show that the latter effect dominates the former in my calibrated model, so a stronger bequest motive lowers the optimal stock share.

I find that the relationship between observed actual household portfolio choice and estimated preferences is qualitatively similar but quantitatively weaker than suggested by the life-cycle model.

To be specific, a stronger preference for LTC expenditure is associated with lower stock share, though the size of the estimated effect is overall much smaller than the predictions from the model. I do not find a significant relationship between the preference for bequests and actual stock share. The discrepancy between the empirical results and the theoretical results might indicate that what households actually do is different from what they should do, which, in turn, suggests a necessity for better design of financial instruments (Campbell, 2006). Using simulated life-cycle profiles from the model solutions, I show that financial instruments need to incorporate implications of the estimated preference heterogeneity not only in determining the initial level of stock market exposure but also in the adjustment of stock share over the life-cycle.

The structure of the rest of the paper is as follows. Section 2 discusses the related literature. Section 3 presents a stylized two-period model to explain the mechanism behind the effect of the health-related risks and the health-state-dependent preferences on the portfolio choice. Section 4 describes the VRI. In Section 5, I explain my methodology of structural preference parameter estimation and present the estimation results. The empirical relationship between household stock share and the preference parameters is discussed in Section 6. In Section 7, I quantitatively examine the implications of the estimated preference heterogeneity on the optimal stock share using a life-cycle model. I discuss the implications of the gap between empirical and theoretical findings in Section 8.

## 2 Literature

The preference parameter estimation in this paper is based on the methodology of Barsky, Juster, Kimball, and Shapiro (1997, hereafter BJKS) and Kimball, Sahm, and Shapiro (2008, hereafter KSS). They estimate the distribution of risk preference parameter using survey responses and allowing for survey response errors. KSS also construct individual cardinal proxies for the risk preference parameters using the estimates from the structural model, which can be used as a right-hand side variable in a linear regression without concerns about an attenuation bias. This paper extends their methodology to the case with multiple preference parameters.

This paper is also related to the literature that estimates health-state-dependent and bequest utility functions. De Nardi, French, and Jones (2010), Ameriks, Caplin, Laufer, and van Nieuwerburgh (2011), Lockwood (2014) and Ameriks, Briggs, Caplin, Shapiro, and Tonetti (2015) estimate preference parameters for health-state-dependent utility functions and/or bequest utility function by either using

a structural model only or combining a structural model with SSQs, but they do not allow for heterogeneity in these preferences. Ameriks, Briggs, Caplin, Shapiro, and Tonetti (2016) estimate heterogeneity in risk preference, precautionary saving motive for LTC, and bequest motive using the SSQs from the VRI. This study differs from theirs in that I examine the impact of preferences on portfolio allocations, while they examine the impact of preferences on demand for LTC insurance.<sup>1</sup> Finkelstein, Luttmer, and Notowidigdo (2009) conclude that using the observed demand for assets with state-dependent returns is the most promising approach in estimating health-state-dependent utility. The SSQs allow us to use this approach.

This paper also adds to the literature that empirically analyzes the effects of health-related risks and bequest motives on households' stock share by distinguishing the role played by preference heterogeneity from that due to other channels. There is a large body of research, including Rosen and Wu (2004), Berkowitz and Qiu (2006), Fan and Zhao (2009), Love and Smith (2010), and Goldman and Maestas (2013), that studies how actual changes in health status (or expected health-related expenditures) affect the stock holdings of households. The literature suggests either no effect or a small negative effect of health-related expenditures on the stock holdings. There is not as much work on the effect of bequest motives on stock share. Hurd (2002) finds no evidence that intended bequests have an effect on stock share, while Spaenjers and Spira (2014) find that households with children tend to have a higher stock share. In most of these empirical studies, the main explanatory variables are remote proxies for (expected) health expenditures and bequests. The remote nature of these proxies makes it challenging to identify the channel behind any observed effect. This paper clearly identifies the effects of preference heterogeneity, controlling for other channels such as effects of different economic and demographic conditions, using responses to the SSQs.

Finally, this paper contributes to the theoretical life-cycle portfolio choice literature by investigating the implications of heterogeneous saving motives. Bodie, Merton, and Samuelson (1992), Viceira (2001), Cocco, Gomes, and Maenhout (2005) and Benzoni, Collin-Dufresne, and Goldstein (2007) use a life-cycle portfolio choice model to analyze the effect of labor income on the optimal stock share. In these papers, retirement is simply considered to be a period without background risk. By contrast, my study provides a more precise understanding of older households

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<sup>1</sup>The estimation methodology in this paper is also different from theirs. I estimate the population distribution of the preference parameters and use the cardinal proxies obtained from the estimated distribution as regressors, following the method outlined in KSS. Ameriks, Briggs, Caplin, Shapiro, and Tonetti (2016) estimate their parameters at the individual level and use them to calibrate their structural model for LTC insurance demand. In Appendix D, I provide a detailed comparison of the two estimation methodologies.

by examining how health-expenditure and mortality risk impact portfolio choices. Ding, Kingston, and Purcal (2014) investigate the effect of a bequest motive on the optimal stock share in the absence of health expenditure shocks and income flow. Pang and Warshawsky (2010) and Reichling and Smetters (2015) study annuity demand using a life-cycle model with exogenous health expenditure risk and bequest motives. This paper solves for the optimal stock share under a life-cycle model that features realistically-calibrated processes for health and income, options for LTC service, and, most importantly, preference heterogeneity estimated from the VRI data.

### 3 A Stylized Two-Period Model

This section presents a stylized two-period model to intuitively illustrate the mechanism behind the effect of health-related risks and health-state-dependent preferences on the portfolio choice of households. In particular, I focus on why households who care to have more resources in an LTC state might want to invest a smaller share of their wealth in risky assets.<sup>2</sup>

Let us assume that the household cares only about the consumption in the second period ( $C_2$ ). Let  $W$  denote the amount of wealth that the household has in the first period. The household invests either in a safe asset, which guarantees gross return of  $R$ , or a risky asset, of which return is  $R + \mu + \eta$  where  $\mu$  is the risk premium and  $\eta$  is the uncertain part of the return. For simplicity, let us assume that  $\eta$  takes value of either  $\xi$  or  $-\xi$  ( $\xi > 0$ ), with a fifty-fifty chance.

The household may or may not need an LTC service in the second period. If it does not need an LTC service, its utility is determined by a log-utility function ( $\log(C_2)$ ); if it does need an LTC service, the utility function becomes  $\zeta_{LTC} \log(C_2 + \kappa_{LTC})$ , where  $\kappa_{LTC} < 0$  determines the subsistence level of expenditure in the LTC state and  $\zeta_{LTC}$  determines the overall strength of the preference for LTC expenditures compared to that for expenditures in the good-health state. Note that these are special cases of more general utility functions that will be introduced in the next section.<sup>3</sup>

Let  $\pi$  be the chance that the household needs an LTC service in the second period. If  $\pi = 0$ , then the household solves:

$$\begin{aligned} & \text{Max}_{\alpha} E_1 \log(C_2) \\ & \text{s.t. } C_2 = (1 - \alpha)WR + \alpha W(R + \mu + \eta) \end{aligned} \tag{1}$$

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<sup>2</sup>The effect of bequest motive on portfolio choice is much subtler and it works through several channels. Discussions on these channels will be presented with a quantitative model in Section 7.

<sup>3</sup>The qualitative results in this section do not depend on the additional assumptions made on the form of utility function in this section. Those assumptions only facilitate deriving a closed form solution.

where  $\alpha$  is the share of wealth invested in the risky asset. Then the optimal share wealth invested in the risky asset,  $\alpha_O$ , is determined as:

$$\alpha_O = \frac{2\mu R}{(\xi + \mu)(\xi - \mu)}. \quad (2)$$

If  $\pi = 1$ , then the household solves the problem that is the same as (1), except for that the LTC-state utility function is used instead of the healthy-state utility function. To simplify the algebra, let us further assume that  $\kappa_{LTC} = -mW$ , i.e., the subsistence level of expenditure in the LTC state ( $mW$ ) is proportional to the household wealth ( $W$ ). The solution in this case,  $\alpha_{LTC}$ , is determined as:

$$\alpha_{LTC} = \frac{2\mu(R - m)}{(\xi + \mu)(\xi - \mu)}, \quad (3)$$

which is apparently smaller than  $\alpha_O$ . The subsistence level of consumption in the LTC state effectively makes the household more risk averse, less willing to take a risk in its financial investment. The larger  $m$  is, the larger the gap between  $\alpha_O$  and  $\alpha_{LTC}$  is.

Now, suppose  $\pi \in (0, 1)$ . The household solves the same problem as (1), except for that the objective function is now  $(1 - \pi)\log(C_2) + \pi\zeta_{LTC}\log(C_2 - mW)$ . The first order condition of this maximization problem becomes:

$$\begin{aligned} & (1 - \pi)\left\{\frac{1}{2}\frac{\mu + \xi}{R + \alpha(\mu + \xi)} + \frac{1}{2}\frac{\mu - \xi}{R + \alpha(\mu - \xi)}\right\} \\ & + \pi\zeta_{LTC}\left\{\frac{1}{2}\frac{\mu + \xi}{R + \alpha(\mu + \xi) - m} + \frac{1}{2}\frac{\mu - \xi}{R + \alpha(\mu - \xi) - m}\right\} = 0. \end{aligned} \quad (4)$$

It is straightforward to show that  $\alpha_O$  makes the first term of the LHS zero, while the second term becomes positive. Similarly,  $\alpha_{LTC}$  makes the second term zero, while the first term becomes negative. Given that the LHS of (4) is continuous and monotonic in  $\alpha$ , the solution for this problem,  $\alpha^*$ , is between  $\alpha_O$  and  $\alpha_{LTC}$ . Finally, as  $\pi$  or  $\zeta_{LTC}$  becomes larger,  $\alpha^*$  gets closer to  $\alpha_{LTC}$ , because the household puts a larger weight on the first order condition derived from the LTC state.

The above example shows that a higher subsistence level of expenditure in the LTC state leads the household to reduce its exposure to financial risk. The effect of the same subsistence level of expenditure becomes stronger when the household puts a larger weight on the utility from the LTC state compared to that from the healthy state. Hence, to fully understand how a household would react to the risk of being in need of LTC in terms of their portfolio choice, we need to estimate

relevant elements in its preferences. I begin to discuss how one can estimate these elements using survey responses in the next section.

## 4 Data

The paper uses the Vanguard Research Initiative (VRI) to estimate the distribution of the structural preference parameters as well as the empirical relationship between households' stock share and heterogeneous preferences for LTC expenditures and bequests. The VRI is a linked survey-administrative dataset on more than 9,000 Vanguard account holders who are at least 55 years old. The VRI is an Internet survey. There have been six surveys to date on different subject areas. This paper uses the first (on wealth and portfolio) and the second (on preferences for LTC and bequests) surveys. Conditional on completing each survey, item non-response rate is very low.<sup>4</sup> 5,741 respondents completed all the questions used in this paper.

This dataset is appropriate for the research question of this paper for several reasons. First, it contains ample observations of wealthholders, for whom LTC precautionary saving motives and bequest motives are operative and financial portfolio choice is a relevant question. Second, the Strategic Survey Questions from the VRI survey allow us to estimate preference parameters. Finally, it includes comprehensive and accurate measures of wealth and stock shares for the account holders. In the following, I discuss each of these features in greater detail.<sup>5</sup>

### 4.1 Sample Composition: Ample Observations of Wealthholders

By design, the VRI collects data on households with non-negligible wealth that are facing key financial decisions in retirement such as annuitization, the purchase of long-term care insurance, or portfolio allocation choices. In contrast, the Health and Retirement Study (HRS), the leading representative survey of older Americans, has a large fraction of households with not enough financial wealth to face such decisions (Poterba, Venti, and Wise, 2011).

The goal of obtaining ample observations of wealthholders is achieved as the VRI is roughly representative of top 50 percent of households in the wealth distribution for the considered age range (55 and above). The sampling screen that is used to target wealthholders—the requirement that they have at least \$10,000 in their Vanguard accounts—made the VRI sample wealthier by construction

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<sup>4</sup>See [http://ebp-projects.isr.umich.edu/VRI/survey\\_overview.html](http://ebp-projects.isr.umich.edu/VRI/survey_overview.html) for more details on the surveys.

<sup>5</sup>See Ameriks, Caplin, Lee, Shapiro, and Tonetti (2015) for complete description of the VRI.



than the more representative samples, such as the HRS and Survey of Consumer Finances (SCF). Ameriks, Caplin, Lee, Shapiro, and Tonetti (2015) show that the VRI is broadly representative of the top half of the wealth distribution and with the similar sampling screens HRS and SCF respondents have similar characteristics as the VRI. In addition, about half of the VRI sample between the ages of 55 and 64 is composed of those who have only employer-sponsored accounts at Vanguard. For this group the selection would be less an issue because they are more likely to have joined Vanguard by their employers' choice, not by their own. Ameriks, Caplin, Lee, Shapiro, and Tonetti (2015) actually show that their characteristics are even closer to those of the comparable subsets of the HRS and SCF. Demographic and economic characteristics of the sample used in this paper are presented in Section 4.4.

## 4.2 Strategic Survey Questions

In its second survey (conducted in winter 2014), the VRI implemented Strategic Survey Questions (SSQs) to elicit information regarding preferences about risk, expenditures on LTC, and bequests. In the following I briefly introduce aspects of the SSQs that are relevant for this paper (see Ameriks, Briggs, Caplin, Shapiro, and Tonetti, 2015, 2016 for a thorough description of these SSQs).

SSQs put respondents in hypothetical situations so that cross-sectional differences in responses can be interpreted as signals of preference heterogeneity. Under hypothetical situations that are not related to their actual financial situations and demographics (including age and health conditions), respondents are asked to choose between hypothetical financial products.

This paper uses three types of SSQs:

- A gamble on consumption to elicit risk preference (SSQ1),
- A trade-off between expenditures in a state of good health versus those in the LTC state, to elicit state-dependent preference for LTC (SSQ2),
- A trade-off between expenditures in the LTC state and bequests to measure the strength of the bequest motive (SSQ3),

The responses to SSQs can be used to identify the preference parameters in the three utility functions, one for expenditures in the healthy state, one for expenditures in the LTC state, and one

for bequests. To do so, I use the fully parametric functional forms:

$$\begin{aligned}
U(X) &= \frac{(X + \kappa)^{1-1/\theta}}{1 - 1/\theta} \\
U_{LTC}(X) &= \zeta_{LTC} \frac{(X + \kappa_{LTC})^{1-1/\theta}}{1 - 1/\theta} \\
U_{Beq}(X) &= \zeta_{Beq} \frac{(X + \kappa_{Beq})^{1-1/\theta}}{1 - 1/\theta},
\end{aligned} \tag{5}$$

where the  $X$  is expenditure in each health state, i.e., good health, LTC state, and bequest/death.  $\theta$  is risk tolerance parameter,  $\zeta$  is a utility multiplier governing the strengths of the precautionary LTC saving and bequest motives, and  $\kappa$  is a necessity parameter for each utility function, determining whether the expenditures are considered necessities or luxuries ( $\kappa$  being negative means the expenditures are necessities, while it being positive means they are luxuries).

In the following, I describe key aspects of setups of each SSQ and distribution of responses. (Table A1 in Appendix A shows the exact setups and wording for each SSQ.) I will also discuss which moment of the SSQ response distributions mainly identifies each preference parameter. I defer more detailed explanation on the modeling of preference heterogeneity, the modeling of survey response errors and the preference parameter estimation procedure to Section 5.

In SSQ1, the key elements of the hypothetical situation are as following. Respondents are at age 80; they live alone and rent their home; it is assumed that they will be healthy for the following year. Respondents have to choose between Plan A and Plan B, where Plan A guarantees a fixed level of consumption ( $\$W$ ) while Plan B has a 50 percent chance of doubling it ( $\$2W$ ) and a 50 percent chance of reducing it by  $x$  percent ( $\$(1 - 0.01x)W$ ) for the following year. Since the question is asked for a sequence of values of  $x$  where the sequence depends on the respondent's previous responses, it provides a risk range ( $[x_{min}, x_{max}]$ ) which encapsulates the respondents indifference point.<sup>6</sup>

Figure 1(a) shows two noticeable patterns in the distribution of responses to the SSQ1. The vast majority of respondents are not willing to take more than 33 percent of risk to have a chance to double their consumption, implying they are overall quite risk intolerant. In addition, when the same question is asked with a relatively lower initial consumption level ( $\$50,000$  instead of  $\$100,000$ ), the results show that respondents tend to be more risk averse. This phenomenon is inconsistent with a homothetic utility function under which we should observe the same response distribution regardless

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<sup>6</sup>The setup of SSQ1 draws on the hypothetical question used in BJKS and KSS, with a key difference that the question used in their papers is about a gamble on income not consumption.

of initial consumption level, motivating the necessity parameter in the healthy-state utility function. In terms of mapping to the preference parameters in the healthy-state utility function, the overall level of risk a respondent is willing to take in SSQ1 identifies  $\theta$  while how much she become more risk averse with a lower initial consumption level identifies  $\kappa$ .

In SSQ2, it is assumed that respondents are still 80 years old but will need a LTC service with probability  $\pi$  in the coming year. There is no publicly-funded LTC service and no one can take care of the respondent for free. They have to allocate the given resources ( $\$W$ ) between Plan C and Plan D, where Plan C pays the respondent  $\$(1/\pi)$  for every  $\$1$  of investment only if the respondent needs an LTC service and Plan D pays  $\$1$  for every  $\$1$  of investment only if the respondent is healthy in the coming year. In the LTC state, respondents need to finance both their LTC expenditures and their other consumption needs out of returns from Plan C. Responses to SSQ2 are measured as the amount of money they choose to invest in Plan C.

Figure 1(b) shows that, in SSQ2, the median respondent allocates resources in such a way that they secure more resources in the LTC state than in the healthy state. For example, when respondents are given  $W = \$100,000$  with an LTC probability of  $\pi = 1/4$ , to have the same amount of resources across two states, they should invest  $\$20,000$  in Plan C. That is indeed close to one of two modal allocations in the distribution, but the majority of respondents invest more than that. This implies level of  $\zeta_{LTC}$  greater than one. How much the share allocated to Plan C changes across different values of  $W$  and  $\pi$  identifies  $\kappa_{LTC}$ .

Lastly, in SSQ3, respondents are assumed to be in the last year of their lives and in need of an LTC service for the entire year. Again there is no publicly-funded LTC service and no informal care. Respondents need to allocate the given resources ( $\$W$ ) between Plan E and Plan F, where money in Plan E will be used to finance their own needs while that in Plan F will be bequeathed. Responses to SSQ3 are measured as the amount of money they choose to put in Plan E.

Figure 1(c) shows that, in SSQ3, when the given resources ( $W$ ) is  $\$100,000$ , many respondents choose not to leave a bequest, but the number of respondents choosing to leave a bequest increases as  $W$  increases. This suggests that bequests may be perceived as luxury goods rather than necessary expenditures (hence  $\kappa_{Beq}$  is positive). Among those who leave bequests, many leave sizeable bequests, implying that once the bequest motive becomes active (i.e., once they have enough resources) it tends to be strong (hence  $\zeta_{Beq}$  is greater than one).

The SSQs have the following common features for eliciting preferences. Each type of SSQ is asked

multiple times with different amounts of given initial resources ( $\$W$ ) and/or with the likelihood of relevant events ( $\pi$ ). In addition to identifying preference parameters as just discussed, this test-retest feature enables us to separately identify the distribution of survey response errors. The SSQ scenarios are also stationary questions embedded in a life-cycle setting. Except for SSQ3 (where it is assumed that respondents die at the end of the following year), it is assumed that the same situation repeats at the end of the following year. Respondents do not have any other resources than what is given in the assumed situations; they are not allowed to either borrow from future or save for future. By shutting down borrowing and lending, responses to SSQs can be interpreted as solutions of single-period maximization problems.<sup>7</sup>

The VRI survey takes a number of steps to make the SSQ scenarios more understandable. In the administration of the survey, respondents are provided with the scenario-specific rules prior to making their decisions. They are also allowed to refer back to the rules via a hover button at any point in the decision process. The survey also tests understanding of scenarios before asking SSQs. A majority of respondents were able to give correct answers to more than 80 percent of the verification questions.

Since the survey is conducted on the Internet, it takes advantage of the ability to visualize the trade-offs of the SSQs. In SSQ2 and SSQ3, participants are asked to make their choices using a novel slider interface (see Figure A1 in Appendix A). This interface dynamically informs participants of the resources they will have in each state as a result of the current allocation.<sup>8</sup>

### 4.3 Wealth and Stock Share Measurement

The wealth and stock share measures are from the first survey of the VRI (conducted in fall 2013). The measures cover the entire financial portfolio and housing wealth of the households.

The wealth and stock share measures of the VRI are based on a comprehensive account-by-account approach. Respondents are asked about the types of accounts they have (e.g., IRAs, savings, mutual funds), the number of accounts of each type, and the balance and stock share of each account. This account-by-account format matches the way respondents keep track of their own wealth and does not require them to sum balances across accounts to provide total figures for asset categories that are

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<sup>7</sup>Note that with constant amount of resources, the life-cycle solution will not involve much borrowing or lending unless the interest rate is different from time discount rate, so the assumption of no borrowing and lending is not drastically counterfactual.

<sup>8</sup>See Ameriks, Briggs, Caplin, Shapiro, and Tonetti (2016) for more detail about verification of rule understanding, the slider interface, and the other mechanisms used for the SSQs. They also show that the SSQ responses are coherent both internally (i.e., there is a strong positive correlation among answers within each SSQ type) and externally (i.e., characteristics that might affect relevant preferences do have predictive power on responses).

familiar to economists but not necessarily to survey respondents.

The accuracy of the wealth and stock measures is validated by comparing them to the Vanguard administrative account records for those accounts they indicate are held at Vanguard. The comparison shows that the survey measure of wealth is very accurate: the median percentage difference between the survey and administrative measures of total assets held at Vanguard is essentially zero while the length of the interquartile range is only several percentage points.<sup>9</sup>

This paper uses the stock share of households' entire financial portfolio measured from the survey as the main dependent variable in the empirical analysis.

#### 4.4 Characteristics of the Sample

In addition to the SSQ responses and wealth measures, I use information on household demographics as well as subjective probability measures regarding their longevity and future need for an LTC service. Table 1 presents the distribution of the variables used in this study beyond the SSQs.

Almost everyone in the VRI is a stock holder, which is not surprising given that the sample is composed of Vanguard clients. Therefore, the identification of any effect on stock holding comes through the intensive margin rather than through the extensive margin, so the interpretation of results of this paper is free from participation cost issues (see Vissing-Jorgensen, 2002). The demographic composition of the VRI sample is as follows. About two-thirds are coupled and two-thirds are male. By design, the VRI respondents are evenly distributed across the following age bins: [55, 59], [60, 64], ..., and 75+. Furthermore, again by design, about half of those below age 65 are from the employer-sponsored sample. Most 401(k) participants roll over to an IRA when they retire, so there are few employer-sponsored accounts for those aged over 65. As a result, about 20 percent of the entire sample are from the employer-sponsored sample. In terms of health, the vast majority of the sample report that their health is better than or equal to good (using a five-point scale *excellent*, *very good*, *good*, *fair*, and *poor*). About 40 percent have a post college degree while another 33 percent have a college degree. The median (mean) household financial wealth and home equity are \$723,665 (\$1,101,468) and \$230,000 (\$354,204). Slightly more than the half of the sample are retired.

As another measure of heterogeneity in household health, the VRI survey asks each participant to estimate her probability of needing at least one year of LTC service as well as her prediction of how likely it is that she will reach a certain age. The results for the VRI sample show that 45 percent

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<sup>9</sup>See Ameriks, Caplin, Lee, Shapiro, and Tonetti (2015) for more details on the wealth measurement in the VRI.

expect to require at least one year of LTC service, with remarkable heterogeneity across responses (the interquartile range is [15%, 75%]). The subjective probability questions about reaching certain ages are asked for a set of ages determined as  $(\{75, 85, 95\} \cap \{t | t \geq \text{current age} + 5\})$ . The median response for the lowest age asked (the measure used in this study) is 85 percent.

Finally, to control for the heterogeneous exposure to LTC expenditure risk caused by LTC insurance, I include the indicator variable of having LTC insurance. Twenty-three percent of the respondents have an LTC insurance policy.

## 5 Estimation of Structural Preference Parameters

In this section I estimate distributions of the structural preference parameters that govern the preferences for LTC expenditures and bequests, based on the methodology of Barsky, Juster, Kimball, and Shapiro (1997, hereafter BJKS) and Kimball, Sahm, and Shapiro (2008, hereafter KSS). The estimates obtained in this section are used to construct the regressors in the empirical analysis in Section 6 and to calibrate the life-cycle model in Section 7.

### 5.1 Methodology

I extend the methodology of BJKS and KSS to estimate joint distribution of multiple preference parameters. I model the respective strengths of the preference for LTC expenditure and that for bequests using a utility multiplier and a necessity parameter for the utility function representing each motivation. Using MLE, I jointly estimate the distributions of these parameters as well as that of the risk preference parameter. Having multiple observations for each type of SSQ enables us also to identify the distribution of the survey response errors.<sup>10</sup> Cardinal proxies for the preference parameters are calculated as the conditional expectations using the estimated distributions. In the following, I explain each element of the estimation methodology in detail.

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<sup>10</sup>To be more specific, to identify distributions of  $N$  parameters in addition to survey response errors, we need at least  $N + 1$  observed responses per individual. In other words, the distribution of SSQ responses should have at least one more degree of freedom than can be explained solely by the distributions of the true preference parameters, to allow room for survey response errors.

**Utility functions** As I previewed in Section 4, I assume three utility functions, one for expenditures in the healthy state, one for expenditures in the LTC state, and one for bequests:

$$\begin{aligned} U_i(X) &= \frac{(X + \kappa)^{1-1/\theta_i}}{1 - 1/\theta_i} \\ U_{LTC,i}(X) &= \zeta_{LTC,i} \frac{(X + \kappa_{LTC,i})^{1-1/\theta_i}}{1 - 1/\theta_i} \\ U_{Beq,i}(X) &= \zeta_{Beq,i} \frac{(X + \kappa_{Beq,i})^{1-1/\theta_i}}{1 - 1/\theta_i}, \end{aligned} \tag{6}$$

where subscript  $i$  denotes each individual.

The necessity parameter ( $\kappa$ ) in the healthy-state utility function has important implications for portfolio choice. Merton (1971) shows that, in a continuous-time model with stock return risk (modeled as an i.i.d. process) as the only uncertainty, for a household with the utility function I assume for the state of good health, the optimal stock share is determined as:

$$\pi = \frac{\mu_s}{\sigma_\eta^2} \theta \left( 1 + \frac{(Y + \kappa)(1 - e^{r(t-T)})}{rW} \right), \tag{7}$$

where  $\mu_s$  is the risk premium,  $\sigma_\eta$  is the standard deviation of the stock return,  $Y$  is the fixed income flow,  $t$  is the current time,  $T$  is the end of the investment horizon, and  $W$  is current wealth. The role of  $\kappa$  is obtained through the ratio between income plus  $\kappa$  and wealth ( $\frac{Y+\kappa}{W}$ ). Intuitively, a higher income-to-wealth ratio should imply a higher optimal stock share, as the present value of the income flow (human capital) becomes a close substitute for a risk-free asset in the absence of income uncertainty.<sup>11</sup> According to (7), what is compared to wealth is not the gross level of income but rather the income net of the subsistence level of consumption (negative of  $\kappa$ ).<sup>12</sup>

**Identification** Here I briefly review which moment of the SSQ response distribution mainly identifies each preference parameter.<sup>13</sup> The risk tolerance parameter ( $\theta$ ) is mainly identified by the

<sup>11</sup>This intuition holds even with income uncertainty as long as income shocks are not highly correlated with stock return shocks (Viceira, 2001; Cocco, Gomes, and Maenhout, 2005).

<sup>12</sup>It should be noted that a negative  $\kappa$  ( $-\kappa$ ) can also be interpreted as (slow-moving) habit in consumption. Habit formation has been used to explain macroeconomic phenomena such as a high risk premium (Campbell and Cochrane, 1999). Both Gomes and Michaelides (2003) and Polkovnichenko (2007) examine the effect of habit formation on household portfolio choices, while Brunnermeier and Nagel (2008) test the microeconomic implications of habit formation. Although I do not explicitly model (the negative of)  $\kappa$  as a time-varying habit, this study provides empirical evidence for this necessity parameter. In a related paper, Wachter and Yogo (2010) explain why more affluent households have a higher stock share using a two-good—basic and luxury—model, where households are less risk averse over luxury good consumption.  $\kappa$  can be considered to be a reduced form representation of this two-good model, since both models generate lower risk aversion for households with larger wealth.

<sup>13</sup>The full relationship between survey responses and preference parameters is complex and non-linear, in particular under the presence of survey response errors. Later in this section, I provide detailed explanations regarding how I

level of risk that respondents are willing to take in SSQ1. The necessity parameter in the healthy state utility function ( $\kappa$ ) is identified by the effect of the initial resource level on the responses in SSQ1. It should be noted that since SSQ1 consists of only two questions, we cannot estimate the distributions of both  $\theta$  and  $\kappa$  and identify survey response errors at the same time. Therefore, I assume that there is heterogeneity only in  $\theta$  (hence no subscript  $i$  for  $\kappa$  in (6)).<sup>14</sup> The utility multipliers for LTC state expenditure ( $\zeta_{LTC}$ ) and bequest ( $\zeta_{Beq}$ ) are mainly identified by the average share of resources that respondents allocate for LTC expenditures and bequests in SSQ2 and SSQ3. The necessity parameters for those two utility functions ( $\kappa_{LTC}$ ,  $\kappa_{Beq}$ ) are mainly identified by how the level of given resources affects responses in SSQ2 and SSQ3.

**Modeling heterogeneity in preference parameters** Following KSS, I model the cross-person heterogeneity of preference parameter as draws from probability distributions. I assume the distribution of the risk tolerance parameter and those of the utility multipliers on LTC expenditure and bequest to be log-normal, while those of the necessity parameters for LTC expenditure and bequest to be normal:

$$\log(\theta_i) \sim N(\mu_\theta, \sigma_\theta^2) \quad (8)$$

$$\log(\zeta_{LTC,i}) \sim N(\mu_{LTC}, \sigma_{LTC}^2) \quad (9)$$

$$\log(\zeta_{Beq,i}) \sim N(\mu_{Beq}, \sigma_{Beq}^2) \quad (10)$$

$$\kappa_{LTC,i} \sim N(\mu_{\kappa,LTC}, \sigma_{\kappa,LTC}^2) \quad (11)$$

$$\kappa_{Beq,i} \sim N(\mu_{\kappa,Beq}, \sigma_{\kappa,Beq}^2). \quad (12)$$

Log-normality assumption prevents the risk preference parameter and utility multipliers from being negative. I also assume that the preference parameters are statistically independent, except for potential dependence through observed covariates.<sup>15</sup>

**Modeling of survey response errors** I model the survey responses as the sum of the solutions of the underlying optimization problems for the SSQs and “trembling-hand” type survey response

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model preference heterogeneity and the survey response process.

<sup>14</sup>When I estimate the distributions of the parameters conditional on the covariates used in the stock share regression, I assume that  $\kappa$  is homogenous conditional on the covariates (i.e.,  $\kappa$  is a deterministic function of these covariates). The motivation for estimating the distributions conditional on the covariates is explained in Section 6.

<sup>15</sup>As will be explained below, in one version of estimation I model the mean ( $\mu$ ) parameter of each of these distributions as a linear function of all the covariates used in the stock share regression. Hence I do allow for correlations between preference parameters through these covariates.



errors, where “trembling-hand” means that error terms are added to the survey responses (instead of preference parameters or utilities). Given the realizations of the preference parameters from (8)-(12), we can determine the solutions of the optimization problems underlying the SSQs. I assume that survey response errors are independent across questions and normally distributed with a mean of zero:

$$\varepsilon_{i,kj} \sim N(0, \sigma_{\varepsilon,kj}), \quad (13)$$

where  $kj$  denotes the  $j^{th}$  question of SSQ type  $k$ .

In the following, I show how to map the SSQ responses to the solutions of the corresponding optimization problems under the presence of the survey response errors.

1) SSQ1: Let  $W_{1j}$  be the amount of consumption given in the  $j^{th}$  question in SSQ1. Given  $\theta_i$  and  $\kappa$ , the level of risk (in terms of the percentage loss associated with the risky gamble) at which individual  $i$  becomes indifferent between the risky gamble and the guaranteed consumption can be determined as  $x_{i,1j}^*$ ,<sup>16</sup> such that:

$$\frac{(W_{1j} + \kappa)^{1-1/\theta_i}}{1 - 1/\theta_i} = 0.5 \frac{(2W_{1j} + \kappa)^{1-1/\theta_i}}{1 - 1/\theta_i} + 0.5 \frac{((1 - x_{i,1j}^*)W_{1j} + \kappa)^{1-1/\theta_i}}{1 - 1/\theta_i}. \quad (14)$$

The indifference point that is actually used in answering the survey question,  $x_{i,1j}$ , is determined as:

$$\tilde{x}_{i,1j} = x_{i,1j}^* + \varepsilon_{i,1j}. \quad (15)$$

This determines the risk range within which the observed response falls.

2) SSQ2: The underlying optimization problem for the  $j^{th}$  question of SSQ2 is:

$$\begin{aligned} \text{Max}_x \quad & (1 - \pi_{2j}) \frac{(W_{2j} - x_{i,2j} + \kappa)^{1-1/\theta_i}}{1 - 1/\theta_i} + \pi_{2j} \zeta_{LTC,i} \frac{(\frac{1}{\pi_{2j}} x_{i,2j} + \kappa_{LTC,i})^{1-1/\theta_i}}{1 - 1/\theta_i} \\ \text{s.t.} \quad & 0 \leq x_{i,2j} \leq W_{2j}, \end{aligned} \quad (16)$$

where  $x_{i,2j}$  is the amount invested in Plan C that pays the respondent when she needs a LTC service and  $\pi_{2j}$  is the likelihood of being in need of a LTC service for the following year. Let  $x_{i,2j}^*$  denote the individual  $i$ 's solution for (16). We can then denote the observed response as

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<sup>16</sup>Note that the unit of  $x$  is share, not percentage.

$\tilde{x}_{i,2j}^{br} = \min(\max(0, x_{i,2j}^* + \varepsilon_{i,2j}), W_{2j})$ , which is the sum of the optimal solution and a survey response error subject to the boundary conditions, where *br* indicates that the response is before rounding. Rounding responses is prevalent in SSQ2 and SSQ3, so in the estimation procedure I need to address the issue of rounding in the estimation procedure. I explain how I do so after introducing the model for SSQ3.

3) SSQ3: The structure for the optimization problem for SSQ3 is similar to that of SSQ2. The underlying maximization problem for the  $j^{th}$  question of SSQ3 is:

$$\begin{aligned} \text{Max}_x \quad & \zeta_{Beq,i} \frac{(W_{3j} - x_{i,3j} + \kappa_{Beq,i})^{1-1/\theta_i}}{1 - 1/\theta_i} + \zeta_{LTC,i} \frac{(x_{i,3j} + \kappa_{LTC,i})^{1-1/\theta_i}}{1 - 1/\theta_i} \\ \text{s.t.} \quad & 0 \leq x_{i,3j} \leq W_{3j}. \end{aligned} \tag{17}$$

The observed response, before rounding, is assumed to be generated through  $\tilde{x}_{i,3j}^{br} = \min(\max(0, x_{i,3j}^* + \varepsilon_{i,3j}), W_{3j})$ , where  $x_{i,3j}^*$  is the solution for (17).

4) Rounding of responses: The distribution of SSQ responses suggests that participants round their answers. For example, Figure 1 shows a bunching of responses at \$100,000 in the second question of SSQ3. This bunching likely reflects rounding since the number of these responses is too high to be generated from smooth distributions of the underlying parameters and survey response errors.

To address this issue, I follow Manski and Molinari (2010) and define the degree of rounding for each respondent using the highest level of precision the respondent provides, separately for SSQ2 and SSQ3. For SSQ2, I set three levels of precision: rounding to multiples of \$25K, rounding to multiples of \$10K, and no rounding. For example, if all of a respondent's answers are multiples of \$25K, then I determine that this is her level of rounding. Then, for this respondent,  $\tilde{x}_{i,2j} = \$50K$  would imply that  $\tilde{x}_{i,2j}^{br} = [\$50K - \$12.5K, \$50K + \$12.5K]$ , with the latter interval used to calculate the likelihood function. If a respondent gives an answer that is neither a multiple of \$25K nor of \$10K, then I assume that she does not round her responses. Using this procedure, I find that 7 percent of respondents round to multiples of \$25K and 8 percent round to multiples of \$10K for SSQ2. For SSQ3, I apply the same logic but allow a higher level of rounding to multiples of \$50K. Doing so, I find that 20 percent of respondents round to multiples of \$50K, 9 percent to multiples of \$25K, and 19 percent to multiples of \$10K.

**Maximum likelihood estimation algorithm** Let  $\tilde{x}_{i,mj}$  be the response observed for the  $j^{th}$  question of SSQ type  $m$ , for individual  $i$  ( $m \in \{1, 2, 3\}$  and  $j \in \{1, 2, 3\}$  ( $j \in \{1, 2\}$  for  $m = 1$ )).

Given the parameter values governing the distribution of the preference parameters and survey response errors,  $\Theta \equiv \{\mu_\theta, \sigma_\theta, \kappa, \mu_{LTC}, \sigma_{LTC}, \mu_{\kappa, LTC}, \sigma_{\kappa, LTC}, \mu_{Beq}, \sigma_{Beq}, \mu_{\kappa, Beq}, \sigma_{\kappa, Beq}, \{\sigma_{\varepsilon mj}\}_{m,j}\}$ , I can calculate the likelihood of having the observed responses in the data. I estimate  $\Theta$  by maximum likelihood estimation. The following summarizes the algorithm.

1. Guess initial values for  $\Theta$ .<sup>17</sup>
2. Given  $\Theta$ , generate  $K$  nodes  $\{\theta^k, \zeta_{LTC}^k, \zeta_{Beq}^k, \kappa_{LTC}^k, \kappa_{Beq}^k\}_{k=1}^K$  in the preference parameter distribution and corresponding probabilities  $\{\pi_k\}_{k=1}^K$  such that  $\sum_{k=1}^K \pi_k = 1$ , using Gaussian Quadrature.
3. For each node  $k$  and individual  $i$ , calculate  $\{\varepsilon_{i,mj,k}^*\}_{mj}$  such that these realizations of the survey response error support the observed responses under  $\{\theta^k, \zeta_{LTC}^k, \zeta_{Beq}^k, \kappa_{LTC}^k, \kappa_{Beq}^k\}_{k=1}^K$ .<sup>18</sup>
4. Calculate the joint likelihood of the realization of the error terms in step 3. If  $\pi_{i,k}^\varepsilon$  denotes this joint likelihood, then:

$$\pi_{i,k}^\varepsilon = \Pi_{m,j} \pi_{i,mj,k}^\varepsilon, \quad (18)$$

where  $\pi_{i,mj,k}^\varepsilon$  is the likelihood of drawing  $\varepsilon_{i,mj,k}^*$ .<sup>19</sup>

5. Calculate the likelihood function for individual  $i$  as:

$$L_i = \sum_{k=1}^K \pi_{i,k}^\varepsilon \pi_k. \quad (19)$$

Then the likelihood function for the entire set of observations is calculated as  $L = \Pi_i L_i$ .

6. Using the Berndt-Hall-Hausman algorithm (Berndt, Hall, Hall, and Hausman, 1974),

<sup>17</sup>I tried various sets of values for initial guesses and found that the estimation results are robust with respect to the initial guess.

<sup>18</sup>In SSQ2 and SSQ3, if the respondent provides an internal response and we determine that she does not round her responses, the corresponding survey response error takes a point value. For all the other cases, including all the cases for SSQ1, the survey response error takes values in an interval.

<sup>19</sup>SSQ1 has two questions while SSQ2 and SSQ3 have three questions. Hence, I weight the likelihood from SSQ1, i.e., I use  $(\pi_{i,mj,1}^\varepsilon)^{3/2}$  in place of  $\pi_{i,mj,1}^\varepsilon$ . Then the likelihood function evenly represents the information contained in each type of SSQ. Intuitively, this weighting scheme is equivalent to assuming that there is a third question in SSQ1 that contains exactly the same information as in the first two questions of SSQ1. Note that the weighting does not have any direct effect on the estimation of parameters related to LTC or bequest preferences, because SSQ1 does not involve these parameters (although it can indirectly affect the estimation of these parameters through the estimation of the SSQ1-related parameters). Without weighting, the estimate for  $\kappa$  is not in line with the pattern we observe in SSQ1, though the identification of that parameter should mainly come from SSQ1. One alternative to weighting is to estimate  $\kappa$  using SSQ1 only, and impose this estimate in the joint estimation. The results for the stock share regression based on this approach are fairly similar to those obtained from the estimation based on weighting.

update the guess for  $\Theta$ . If the new guess is sufficiently close to values assumed in step 1, stop. Otherwise go back to step 2 with the updated values.

**Construction of the Cardinal Proxies for the Preference Parameters** Once I obtain the estimates  $\hat{\Theta}$ , I then calculate the cardinal proxies for the preference parameters conditional on observed responses using Bayes's rule:

$$E[\Xi_i | \hat{\Theta}, \{\tilde{x}_{i,mj}\}_{m=1,2,3;j=1,2,3}] = \frac{\sum_{k=1}^K \Xi^k \pi_k \pi_{i,k}^\varepsilon}{L_i} \quad (20)$$

for  $\Xi \in \{\log\theta, \log\zeta_{LTC}, \log\zeta_{Beq}, \kappa_{LTC}, \kappa_{Beq}\}$ , where  $\Xi^k$  is the value of  $\Xi$  that corresponds to the  $k$ -th node of the Gaussian quadrature.<sup>20</sup> Note that when I estimate the distributions conditional on the covariates used in the stock-share regression, the same algorithm applies, but the means of the preference parameter distributions  $\{\mu_\theta, \mu_{LTC}, \mu_{\kappa,LTC}, \mu_{Beq}, \mu_{\kappa,Beq}\}$  and  $\kappa$  are modeled as linear functions of those covariates.

## 5.2 Estimation Results

In this section, I present the results of the estimation. The results in Table 2 show the estimated distributions of the preference parameters and survey response errors. (Appendix B shows the estimates conditional on the covariates.) Panel (a) of Table 2 shows the estimated moments of the distributions while Panel (b) shows the distributions of the preference parameters implied by these moments.

The necessity parameter for the healthy-state utility function,  $\kappa$ , is estimated to be  $-\$10.82K$ , implying that  $\$10.82K$  per year is the subsistence level of consumption. The interquartile range for the risk tolerance parameter is  $[0.17, 0.43]$ , which can be translated into a relative risk aversion of  $[2.70, 6.67]$  ( $[3.03, 7.69]$ ) under the estimated  $\kappa$  and a consumption level of  $\$100K$  ( $\$50K$ ). Although this range is slightly lower than the interquartile range from the KSS estimates ( $[3.84, 10.00]$ ) that are obtained from the entire HRS sample, the difference is small.

The fact that the VRI and HRS have similar distributions of risk preference has the following two important implications. First, it suggests that low stock holdings in the HRS is mainly due to low wealth level or other economic factor, not different risk preference than found in the VRI. Second,

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<sup>20</sup>For parameters that I assume to be log-normally distributed, I use log of the parameters in the empirical analysis in Section 6.

it suggests that the findings from the VRI sample can be extrapolated to the population because risk preference is similar to that found in a representative population despite the higher wealth and education of the VRI population.

Furthermore, we see from the estimation results that there is a substantial heterogeneity in each of the utility multipliers. At the 10th percentiles, respondents do not put much weight on LTC expenditures/bequests ( $\zeta_{LTC} = 0.19$ ,  $\zeta_{Beq} = 0.12$ ) compared to expenditure in the healthy state. By contrast, at the 90th percentiles, respondents place great weight on these expenditures ( $\zeta_{LTC} = 70.91$ ,  $\zeta_{Beq} = 1134.69$ ). For respondents with these preference parameter values, the importance of expenditure in the healthy state is dwarfed by the importance of these expenditures.

For the necessity parameters of LTC-state and bequest utility functions ( $\kappa_{LTC}$  and  $\kappa_{Beq}$ ), the mean of the former is smaller than  $\kappa$  and that for the latter is larger than  $\kappa$ , implying that the average respondent considers LTC expenditures as a necessity and bequests as a luxury in comparison to spending in the state of good health. But there are also strong heterogeneities in both of these parameters. The interquartile ranges are [-52.61K, -19.33K] for  $\kappa_{LTC}$  and [8.32K, 69.16K] for  $\kappa_{Beq}$ . There are also some households that consider expenditures in the LTC state as a luxury good compared to spending in the state of good health (i.e.,  $\kappa_{LTC} \geq \kappa$ ), as well as some households that consider bequests as a necessity compared to spending in the healthy state (i.e.,  $\kappa_{Beq} \leq \kappa$ ).

Given that both the utility multipliers and the necessity parameters govern the respective strengths of the saving motivations, just looking at the distribution of each parameter separately is not enough to understand the degree of heterogeneities in these motivations. To show the implications of the estimated distributions of the parameters more clearly, in Appendix C I solve a static optimization problem for a one-year period where each household allocates its given wealth into expenditures in the healthy state, expenditures in the LTC state, and bequest and present the distribution of these allocations.

## 6 The Empirical Relationship between Stock Share and Saving Motives

How does the estimated strong heterogeneity in preferences relate to the actual portfolio choices of households? In this section I answer this question by relating stock share to SSQ responses, both using the raw SSQ responses as a reduced form analysis and the cardinal proxies for the preference

parameters as a structural analysis.

## 6.1 Analysis Using Raw SSQ Responses

For the reduced form analysis, I define the SSQ1 regressor based on categories of how much risk the respondent is willing to take to have a 50 percent chance of doubling her consumption, with  $W_1 = \$100K$  (the most risk averse category, i.e., 0-10%, is the omitted category). For SSQ2, I use the fraction of wealth that the respondent allocates to the LTC state (averaged over three questions). Finally, for SSQ3, I use the share of wealth bequeathed (averaged over three questions).

The results in Table 3 show a statistically significant relationship between the proportion of stock in a households portfolio and the raw responses to SSQ1 and SSQ2. Specifically, I find that risk tolerance is positively related to the stock share. Willingness to take the risk of losing 33-50% of consumption, compared to 0-10%, increases the stock share by 6 percentage points (5 percentage points after controlling for the covariates). I also find that the willingness to allocate more resources to LTC expenditures, proxied by SSQ2, is negatively related to the stock share; this result becomes only marginally significant at the 10% level when I control for covariates. Giving 10% more of wealth to the LTC state in SSQ2 is associated with an approximate 0.5 percentage point (0.3 percentage point when using the covariates) decrease in the stock share. Finally, I find no significant relationship between the willingness to bequeath, proxied by SSQ3, and the stock share, with point estimates close to zero.

While these results are indicative of the effect of preference heterogeneity on stock share of households, the use of raw responses limits analysis due to attenuation bias caused by survey response errors as well as the difficulty in quantitatively interpreting the regression results without mapping the SSQ responses to the structural preference parameters. For these reasons, I turn to analyses using the cardinal proxies for the preference parameters.

## 6.2 Analysis Using Cardinal Proxies

The cardinal proxies are generated regressors. Nonetheless, we can still obtain unbiased estimates using them. If the difference between a generated regressor and the true variable is a classical measurement error then using the generated regressor instead of the true variable yields an attenuation bias. The cardinal proxies for the preference parameters constructed under the estimation methodology of this paper, however, are free from this issue. They are calculated as conditional

expectations, so by construction, the difference between the latent variables and the proxies is uncorrelated with the proxies. Appendix D extends this discussion.

Table 4 shows the results from analyses using the cardinal proxies for the preference parameters. Specification 1 includes only the cardinal proxies while Specification 2 also includes the control variables. For Specification 2, I use the cardinal proxies from the structural preference parameter estimations conditional on these same controls. As stressed by KSS, when the preference parameter proxies are to be included as a regressor in an equation of interest, the proxies must be constructed conditional on all the covariates in the question of interest. Otherwise, deviation of the proxy from its true value will be correlated with covariates, which biases the coefficient estimates in the equation of interest.

These analyses yield qualitatively similar results to those using the raw responses. That is, I again find that risk tolerance is positively correlated with the stock share while the utility multiplier for the LTC state is negatively correlated with the stock share. The relation between risk tolerance and the stock share is similar to what theoretical models predict (see (7) for example). For LTC expenditures, given the same probability of being in need of a LTC service, a larger  $\zeta_{LTC}$  implies a larger effective size of the expenditure shock associated with this health shock, reducing the stock share of households as predicted by the two-period model in Section 3. Finally, I find that neither the necessity parameter for the LTC state ( $\kappa_{LTC}$ ) nor the multiplier ( $\zeta_{Beq}$ ) or necessity parameter ( $\kappa_{Beq}$ ) for the bequest utility has a consistently significant effect on portfolio composition.

Since the coefficients in Table 4 do not clearly show the quantitative implications of preference heterogeneity on portfolio composition, I calculate the implied change in the stock share when each cardinal proxy is increased by two standard deviations and present the results in Table 5. Two-standard-deviation increase in  $\theta$  increases the stock share by about 3.7 (3.1 with controls) percentage points. The increase yields a slightly larger effect for  $\zeta_{LTC}$ , decreasing the stock share by about 4.6 (4.0 with controls) percentage points. The other parameters yield smaller effects and as we have seen in Table 4 these effects are not consistently statistically significant.

### 6.3 Robustness Checks: Investigations on the Impact of Housing Wealth

In the baseline empirical analysis the linear effect of the log of home equity on the level of stock share is controlled, but this specification might not be able to fully account for the impact of home equity on the portfolio choice of households. To be specific, households may consider home equity as a direct

substitute for risky financial assets, or as a direct substitute for safe financial assets, depending on how they view their home equity.

To address this concern, I examine how the result changes by including home equity as a safe asset and a risky asset in the calculation of the dependent variable. Column 2 and 3 in Table 6 report the regression results where the left-hand-side variable, the share of risky assets, includes home equity as a safe asset and a risky asset, respectively. Compared to the baseline result, which is reported again in Column 1 of Table 6 for convenience, the magnitude of the estimated coefficients are slightly smaller but all the patterns are similar. Hence considering housing wealth as a direct substitute for risky financial assets or safe financial assets does not alter the main empirical findings. This result is not surprising given that for this sample home equity on average accounts for only a small fraction of their entire wealth.

## 7 Life-cycle Portfolio Choice Model with Multiple Late-in-Life Saving Motives

To investigate the theoretical implications of heterogeneous preferences on the optimal stock share of a household's portfolio, I build a life-cycle portfolio choice model featuring the preference heterogeneity estimated from the VRI. Overall, the results from the model are qualitatively in line with the empirical results. But I also find that in some cases the theoretical model predicts larger quantitative effects of the heterogeneous saving motives on portfolio choices than those found in the empirical analysis.

### 7.1 Model

In the life-cycle portfolio choice model, households are subject to aggregate stock market return risk as well as idiosyncratic health, mortality and labor income risks. In each period in the model, households must determine how much to save and consume and how much of their savings to be invested in stocks. Households in an LTC state must also determine whether to use a private LTC service after paying costs out of pocket or a means-tested, publically-funded LTC service after forfeiting the entire wealth. The amount of LTC expenditures in the case of using a private LTC service is endogenously determined, based on the utility function for the LTC state. Any wealth remaining at the end of life is assumed to be bequeathed. To assess the effect of heterogeneous saving motives on the optimal stock share, I compare the policy functions across individuals who differ only in their preference parameters.



**Health transitions and preferences** In this model, a household is composed of a single member.<sup>21</sup> The model starts from age 55, which is the lowest age observed in the VRI, and the household can live up to age 110. Each period, the health status ( $s$ ) of a household takes one value from the set  $\{G, B, LTC, D\}$ , where each state means good health, bad health, LTC state, and death, respectively. The health state evolves following a first-order Markov process with the transition matrix  $\pi_{ss'}$  where  $D$  is an absorbent state. The transition matrix  $\pi_{ss'}$  is also a function of age ( $t$ ) and gender ( $g$ ). Households discount the next period utility by the time discount factor  $\beta$ .

In this model, the utility function depends on the health state, as specified in (2). When  $s = G$  or  $B$ , the utility function is  $U_i$ , while with  $s = LTC$  it becomes  $U_{LTC,i}$ . In the LTC state, the (subjective) minimum required LTC expenditure is captured in the negative of the necessity parameter ( $-\kappa_{LTC,i}$ ). The amount a household chooses to spend on LTC service in addition to  $-\kappa_{LTC,i}$  depends on other parameter values, in particular  $\zeta_{LTC,i}$ .<sup>22</sup> In the LTC state, a household also has the option of using a publicly-funded LTC service after forfeiting all of its wealth. In this case, the value of the public LTC service expressed in the expenditure equivalence is parameterized as  $PC$ , so that the corresponding utility becomes  $U_{LTC,i}(PC)$ .<sup>23</sup> When a household draws  $s = D$  for the first time, it leaves all its wealth as bequests, with the utility determined by  $U_{Beq,i}$  and no utility obtained in subsequent periods.

**Labor income process** The model assumes that a household retires at age 65. Until then, its labor income is exogenously determined as:

$$\log(Y_{it}) = \log(\bar{y}_i) + \nu_{it}, \quad \nu_{it} \sim N(0, \sigma_\nu^2), \quad \forall t < 65, \quad (21)$$

where  $\bar{y}_i$  is the mean income before retirement and  $\nu_{it}$  is a temporary shock. Given that households have only 10 years until retirement in this model, I abstract from permanent income shocks. After retirement, a household receives a retirement income that captures both Social Security income and a defined benefit pension income and hence comes with no uncertainty. This annuity income is modeled

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<sup>21</sup>This is to avoid complications arising from modeling joint survival probabilities and spousal benefits for Social Security and defined benefit pension plans. The estimated relationship between the stock share and preference parameters is not appreciably different between single households and coupled households.

<sup>22</sup>Other than  $-\kappa_{LTC,i}$  and  $-\kappa$ , I do not explicitly model mandatory and uninsured health cost.

<sup>23</sup>I do not explicitly model welfare in the other health states given that the sample is affluent enough to finance expenditures of at least  $-\kappa$  every period. In the model, the lowest support for the income process is set to be larger than  $-\kappa$  for all the ages considered.

as a fraction ( $\lambda$ ) of the mean income before retirement:

$$\log(Y_{it}) = \log(\lambda) + \log(\bar{y}_i), \forall t \geq 65. \quad (22)$$

**Financial Assets** Households can invest in two different assets: a riskless asset and a risky asset, where the latter represents stocks.<sup>24</sup> The gross real return on the risk-free asset is set as the constant  $\bar{R}_f$ . The distribution of the real gross return on the risky asset,  $R_t$ , is modeled as:

$$R_t = \mu_s + \bar{R}_f + \eta_t, \eta_t \sim N(0, \sigma_\eta^2), \quad (23)$$

where  $\mu_s$  is the risk premium and  $\eta_t$  is an i.i.d. stock return shock. Following Cocco, Gomes, and Maenhout (2005), I assume that the aggregate stock return shock is uncorrelated with the temporary labor income shock.

**Optimization problem of households** To specify the optimization problem, I begin by letting  $W_{it}$  represent the beginning-of-period financial wealth of a household, and  $\alpha_{it}$  be the share of savings invested in stocks, with  $Gov_{it}$  indicating whether a household chooses to use a publicly-funded LTC service in the LTC state ( $Gov_{it} = 1$  means it uses a publicly funded LTC service, while  $Gov_{it} = 0$  means it purchases a private service). The optimization problem, omitting the subscripts  $i$  and  $t$ , can then be written as:

$$\begin{aligned} V(W, t, s, g) = & \max_{X, W', \alpha, Gov} I_{s=LTC}(1 - Gov)U_{LTC}(X) + I_{s=LTC}GovU_{LTC}(PC) + I_{s=G,B}U(X) \\ & + \beta E\left[\sum_{s'=G,B,LTC} \pi_{ss'}(t, g)V(W', t+1, s', g) + \pi_{sD}U_{Beq}(W')\right] \\ s.t. & W' = (1 - Gov)[(W - X)((1 - \alpha)\bar{R}_f + \alpha R_s)] + y', X \leq W, \alpha \in [0, 1]. \end{aligned} \quad (24)$$

Note that I do not allow borrowing or the short-sale of stocks; hence the last constraint is imposed.

**Computation** I solve for the optimal policy function numerically using backward induction. Since the last period maximization problem is static the value function is trivially obtained. This value function is used as a continuation value for the maximization problem of the penultimate period. I repeat this until the maximization problem at the first period is solved.

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<sup>24</sup>I abstract from housing wealth in this model.

For the choice over continuous spaces (i.e., over  $X$  and  $\alpha$ ), the optimization is done using a grid search. Normal distributions for labor income and stock return risks are approximated as discrete processes using a Gaussian quadrature.

**Calibration** To understand the effects of heterogeneous preferences on the optimal stock share of a household’s portfolio, I solve the model for various sets of preference parameter values that reflect the range of estimates in the VRI. I focus on the effect of being one standard deviation away (both up and down) from the mean of each preference parameter distribution, so that I can compare the effect of two-standard-deviation difference in each preference parameter to the results in Table 5. The necessity parameter for the ordinary utility function ( $\kappa$ ) is fixed at the value estimated from the VRI ( $-10.82K$ ). The time discount factor ( $\beta$ ) is set at 0.96, a value typically used in the literature for annual models. I calibrate the value of a publicly-funded nursing home to be equivalent to that of spending slightly more than the subjective minimum expenditure on a private LTC service ( $PC = -\kappa_{LTC} + 10K$ ).<sup>25</sup>

The health transition Markov process matrix  $\pi_{ss'}$  is estimated from the sample from the HRS (2002-2014) that satisfy the VRI sampling criteria. Since the HRS is a biennial survey, I first estimate a multinomial logit model for the biennial transition process conditional on age, gender, and current health status, and then transform the estimated process into annual one. See Appendix E for a detailed explanation of this estimation process.

The calibration of asset returns is mainly based on Cocco, Gomes, and Maenhout (2005). A risk-free return ( $\bar{R}_f$ ) is set at 1.02. The standard deviation of the risky asset return ( $\sigma_\eta$ ) is set at 0.17. Since the focus of this paper is to analyze the effects of different saving motives at the intensive margin rather than solving the risk premium puzzle, I set the risk premium ( $\mu_s$ ) to be lower (0.025) than the 0.04 used in Cocco, Gomes, and Maenhout (2005).<sup>26</sup> <sup>27</sup> With this risk premium, the stock share in the model around the median level of financial wealth observed in the VRI (about \$700K) is close to the mean value in the VRI (about 0.55).

I use three values for the mean annual income before retirement ( $\bar{y}$ ): \$45K, \$90K, and \$120K for

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<sup>25</sup>Note that, at the estimated median of  $\kappa_{LTC}$ , this value of  $PC$  becomes close to what Ameriks, Briggs, Caplin, Shapiro, and Tonetti (2016) estimate using a SSQ not used in this paper.

<sup>26</sup>When  $\mu_s = 0.04$ , for the range of the risk preference parameter estimated from the VRI, households choose to invest their entire savings in stocks, not allowing variations at the intensive margin. Cocco, Gomes, and Maenhout (2005) still obtain an interior solution with this risk premium by calibrating the relative risk aversion at 10. However, this value is out of the range supported by the VRI estimates.

<sup>27</sup>A lower risk premium can be considered a reduced form representation of ambiguity aversion among households with respect to the mean of stock return distribution. In addition to the risk modeled in (18), if there is additional uncertainty (ambiguity) about the value of  $\mu_s$ , and if respondents have aversions to this ambiguity, their portfolio choice should resemble that of those who believe  $\mu_s$  to be low without ambiguity (see Klibanoff, Marinacci, and Mukerji, 2005).

those who are still working. These values are the median and the interquartile income distribution range in the VRI. To calibrate the replacement rate after retirement ( $\lambda$ ), I calculate the ratio between the expected annuity income (Social Security income plus the defined benefit pension income) and the current household income for those who are still working. The parameter is calibrated to the mean of the distribution of this ratio, 0.5. Finally, the transitory income shock variance ( $\sigma_\nu^2$ ) is set at 0.07, close to the value used in Cocco, Gomes, and Maenhout (2005).<sup>28</sup>

Table 7 outlines the parameter calibration. Panel A summarizes the values used for the heterogeneous preference parameters while Panel B shows the other parameter values.

## 7.2 Results

By comparing policy functions across households with different preference parameters, I first find that both a stronger precautionary saving motive for LTC and a stronger bequest motive lower the optimal stock share. I then investigate the mechanism behind these effects by shutting off some risks in the model. I also find that the slope of the life-cycle profile of stock share depends on the strength of each saving motivation.

### 7.2.1 Effect of Preference Heterogeneity on the Optimal Stock Share

To put the results in the context of the literature, I first investigate how the optimal stock share changes over income, wealth, and age for households with the median values for all the preference parameters. Figure 2 shows the stock share policy functions for males in good health with median preferences. Panel (a) is for age 55, while (b) is for age 80.

The main driving force behind the differences in the optimal stock share in this figure is the ratio between a household's financial wealth and the value of human capital, where the latter is a present value sum of labor and retirement income. When there is no risk in retirement income, and when labor income risk is not correlated with stock returns, human capital functions as a close substitute for risk-free assets.<sup>29</sup> In this case, a household with relatively more human capital should have a higher stock share in its financial portfolio. Hence, higher wealth should be associated with a lower stock share given income levels, while higher income and a younger age should be associated with a higher

<sup>28</sup>They estimate this to be 0.058 for college graduates. I set it slightly higher here given that my model does not have permanent income shocks.

<sup>29</sup>Viceira (2001) shows that this is still the case even with moderate correlation between labor income and stock return processes.

stock share given wealth levels. This is the mechanism that Cocco, Gomes, and Maenhout (2005) and Viceira (2001) focus on.

Now I investigate the effects of the preference heterogeneity. Figure 3 shows how the optimal stock share changes when we increase risk tolerance and decrease the strength of each saving motive (i.e., decrease each utility multiplier and increase each necessity parameter), for age 55 and selected combinations of wealth and income. When I analyze the effect of one preference parameter, the other parameters are set at the median values. The changes reflect the effects of two-standard-deviation changes in the preference parameters for ease in comparing them to the empirical estimates in Table 5.

Qualitatively, the results for the effect of the risk tolerance and the preference for LTC expenditures are similar to what I find from the empirical analysis. Being more risk tolerant increases the optimal stock share, while having a higher  $\zeta_{LTC}$  implies a lower stock share, as in the empirical analysis. For the other parameters, I find patterns that are not found in the empirical analysis. A lower  $\kappa_{LTC}$  has an effect similar to a higher  $\zeta_{LTC}$ : when the subjective minimum requirement expenditure in the LTC state is higher, the optimal stock share is lower. The effects of both  $\zeta_{Beq}$  and  $\kappa_{Beq}$  show that a stronger bequest motive is associated with a lower stock share.

For most of the parameters, the model predicts greater effects than found in the actual behavior of the VRI sample. Increasing risk tolerance by two standard deviations in the model is associated with a more than 40 percentage point increase in the stock share across wealth and income levels, compared to the 3.2 percentage point increase found in the empirical analysis in Section 6. The heterogeneity in the other parameters have smaller but still substantial effects. For example, heterogeneity in precautionary saving motives for LTC, expressed as differences in  $\zeta_{LTC}$ , creates about a 4 percentage point difference in the stock share on average, while the difference can be as large as 8 percentage points.<sup>30</sup> The corresponding numbers for  $\kappa_{LTC}$  are somewhat larger—6 percentage points on average and more than 12 percentage points for some cases. Note that the numbers from the empirical analysis were similar in the case of  $\theta_{LTC}$  (4.0 percentage points), while for  $\kappa_{LTC}$  it was much smaller (2.1 percentage points) and the sign was wrong. Finally, the effects of heterogeneity in  $\zeta_{Beq}$  and  $\kappa_{Beq}$  are smaller compared to that of  $\zeta_{LTC}$  and  $\kappa_{LTC}$ , but in many cases they are still larger than the numbers from the empirical analysis and for  $\zeta_{Beq}$  the direction is actually opposite to what the point

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<sup>30</sup>When the income level is high and the wealth level is low, the effect is zero, since for these households, under the range of values of  $\zeta_{LTC}$  used in this analysis, the optimal stock share is 100 percent. Large effect of heterogeneity in  $\zeta_{LTC}$  for these households will be obtained if I allow for leveraging. The same caveat applies to the analyses for the other preference parameters.

estimates from the empirical analysis suggest.

I find almost the same pattern for age 80 (Figure 4). In many cases the size of the effect is reduced in terms of percentage point differences, but this is mainly due to that older households have a lower stock share than younger households because of the reduced value of human capital (see Figure 2(b)). In terms of percent difference (not percentage point difference) in stock share, the effects have similar magnitudes. Notice that the effects of  $\kappa_{LTC}$  and  $\kappa_{Beq}$  are much larger for the older households. As households get older, they tend to have a lower level of wealth and income, so the effects of the necessity parameters of which the units are fixed in terms of dollars get larger.

### 7.2.2 Mechanisms Behind the Effects

**Negative impact of a stronger LTC precautionary motive on the optimal stock share** A higher  $\zeta_{LTC}$  or lower  $\kappa_{LTC}$  means that households face larger background risk, since an endogenously-determined LTC expenditure is larger when households are hit by an LTC shock. Therefore, those with higher  $\zeta_{LTC}$  or lower  $\kappa_{LTC}$  want to reduce their exposure to financial market risk, as shown in Section 3.

For those who do not have enough resources, the availability of publicly-funded LTC service reduces this effect. This fact is well demonstrated in the effect of  $\kappa_{LTC}$ . For the low-wealth and low-income combination, larger required expenditures in the LTC state (i.e., lower  $\kappa_{LTC}$ ) is associated with a higher optimal stock share (see Figure 3(e) and 4(e)). When both the wealth and income levels are low, and if they are going to spend significant money on LTC service when they are hit by a LTC shock, it is more likely that they will end up using the option of a public LTC service. Consequently, this household would be less affected by the combination of a negative stock return shock and an LTC shock since it would forfeit its wealth anyway upon choosing to use the public LTC service.

**Negative impact of a stronger bequest motive on the optimal stock share** The negative effect of a stronger bequest motive on the optimal stock share, in particular that of  $\zeta_{Beq}$ , may seem puzzling given that the medium value of  $\kappa_{Beq}$  is positive. Since a bequest is a luxury good, higher weight on the bequest motive should imply lower effective risk aversion. Ding, Kingston, and Purcal (2014) confirmed this in an environment without health and mortality risks and income.

The negative effect comes from the two elements of the model: the existence of retirement income and LTC risk under the presence of mortality risk. First, to understand the role played by the

retirement income, suppose that a household that invested its entire wealth in stocks experiences a negative ten percent stock return. If that household has mainly been saving to finance its own consumption rather than to bequeath its wealth, this loss of stock value will not translate into a ten percent reduction in permanent consumption as long as the household has significant retirement income from either Social Security or defined benefit pensions, which is not affected by the stock market performance. If that household has mainly saved to leave bequests, however, the loss in stock value can be translated into about a ten percent reduction in bequests, in particular when the household dies soon after that, because unrealized retirement income cannot be bequeathed. In short, the existence of unrealized retirement income and mortality risk can increase the effective risk of a negative stock market return for those with stronger bequest motives.<sup>31</sup>

Second, the effective risk of the same LTC shock is larger for a household with larger  $\zeta_{Beq}$  because when a household is hit by a LTC shock, the amount of wealth that can be bequeathed is dramatically reduced and, at the same time, mortality risk is increased. They would not have enough time to accumulate their wealth again until they die. For those who mainly care about their own consumption (i.e., those with lower  $\zeta_{Beq}$ ), however, the fact that an LTC shock accompanies the increase mortality risk is functioning as an insurance since the chance that they will outlive their resources is reduced with the higher mortality risk.

To measure the effect of an LTC shock on bequests, I ran 10,000 simulations for each value of  $\zeta_{Beq}$  considered and calculated the average bequest conditional on the age at death,  $\zeta_{Beq}$ , and also on whether the household ever had an LTC shock in its lifetime or not (see Section 7.3 for details on the setup of the simulations). Figure 5(a) shows the result. (Figure 5(b) plots the survival rate up to each age to show the likelihoods of dying at different ages.) Having an LTC shock in their lives reduces bequests on average about \$150,000 for all the  $\zeta_{Beq}$  values and for most of ages at death. In terms of proportion rather than absolute value, the size of the shock on bequest gets larger as they die at higher ages. And this shock is more painful to those with high  $\zeta_{Beq}$  values.

To test the strength of these channels, I shut off LTC and mortality risks (i.e., all households live up to 110 years old and do not need LTC) and revisit the effect of  $\zeta_{Beq}$  (Figure 5(c)). In the absence of these risks, we see that the stronger bequest motive implies higher stock share, as in Ding, Kingston, and Purcal (2014), though the effect is in general very small and it becomes essentially null

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<sup>31</sup>It is clear that the size of this effect should depend on the replacement rate ( $\lambda$ ) of retirement income. Hence, we can predict that the transition from a defined-benefit to a define-contribution pension system should reduce the effect of this channel.

for households with high wealth.

**Small effect of bequest motive** Though a strong bequest motive lowers the optimal stock share, overall the size of the effect is not large. To investigate the reason for this small effect, in Figure 6 I separately calculate the effect of one-standard-deviation differences of  $\zeta_{Beq}$  in its lower range (low  $\zeta_{Beq}$  - medium  $\zeta_{Beq}$ ) and higher range (medium  $\zeta_{Beq}$  - high  $\zeta_{Beq}$ ). The figure shows that the effect of  $\zeta_{Beq}$  is non-linear: the effect is almost null until it becomes large enough. Almost the entire effects of two-standard-deviation differences in  $\zeta_{Beq}$  are from the one-standard deviation differences in its higher range. Since the median LTC precautionary saving motive is already strong enough to induce sizeable accidental bequests as long as they do not live up to very high ages and/or they are not hit by a long sequence of LTC shocks, when the bequest motive is not strong, it is already saturated with these accidental bequests. This can be seen from the fact that one-standard-deviation difference in  $\zeta_{Beq}$  in its lower range does not affect the accumulation of wealth and hence in the amount of accidental bequests (Figure 5(a)). Once  $\zeta_{Beq}$  is large enough, it starts to affect both wealth accumulation and portfolio allocation.

### 7.3 Life-Cycle Profile of Stock Share

To investigate the model's implications for the design of financial advice over life-cycle, I generate life-cycle profiles of stock share by simulating the model. Here I focus on different values of  $\zeta_{LTC}$  and  $\zeta_{Beq}$  considered in Figure 3 and 4. For each parameter value, I simulate 10,000 times using the policy functions for saving and portfolio choices and take the average to obtain the profiles. The simulation starts from age 55 with a wealth level of \$700K. I set  $\bar{y}$  to be \$90K. In Figure 7, I show profiles across different  $\zeta_{LTC}$  (Panel (a) and (b)) and  $\zeta_{Beq}$  (Panel (c) and (d)).

In Panel (a) I first show the average wealth profile since different preference parameter values affect wealth-to-income ratio and the latter, in turn, affects the optimal stock share. Households accumulate wealth before retirement (age 65) and then decumulate afterward. With a stronger precautionary saving motive associated with LTC (i.e. higher  $\zeta_{LTC}$ ), the accumulation is faster while the decumulation is slower, leading to an overall higher wealth level.

Accumulation of wealth and approaching retirement together can explain the downward sloping stock share profile before retirement depicted in Panel (b). In this phase, the slope is the same across different values of  $\zeta_{LTC}$ . Furthermore, the profile in this phase is qualitatively consistent with and



quantitatively not very different from the often-mentioned rule of thumb for life-cycle funds, which says that stock share in terms of percentage should be determined by subtracting one's age from 100.

After retirement, however, the slope depends on the strength of the precautionary saving motive. At the median preference, the slope becomes flat after retirement. With a strong precautionary saving motive for LTC, the slope is much more negative; the slope becomes positive when this saving motive is weak. Differences in the slopes, again, mainly reflect differences in the wealth-to-human-capital ratio across different groups. Those with a stronger precautionary saving motive for LTC save more, and an increased wealth-to-human-capital ratio implies a lower stock share.

Panel (c) and (d) show that I obtain similar results over  $\zeta_{Beq}$ . One noticeable difference is that until  $\zeta_{Beq}$  becomes large enough, the effect of that preference parameter on both wealth accumulation and portfolio allocation is negligible, given the strength of the median precautionary saving motive for LTC.

This exercise shows that there is no uniform rule for stock share adjustment over the life-cycle that can be applied to every household. The rules, on the one hand, should consider differences in the optimal stock share, given wealth level, across households with different motivations for saving (reflected in different initial levels of stock share profiles in Figure 7(b) and 7(d)), and on the other hand, the rules should also consider different wealth-to-human capital ratios that result from the heterogeneity in motivations for saving (reflected in different slopes of stock share profiles in Figure 7(b) and 7(d)).

## 8 Conclusion

I find evidence that both the preferences for LTC expenditures and bequests are overall strong but also heterogeneous across households. The former implies that older households are on average substantially exposed to health-related expenditure risks and mortality risk, while the latter implies that there is large heterogeneity in their exposures to these risks. The life-cycle portfolio choice model with the estimated preference heterogeneity predicts that the optimal stock share is lower for a household with either a stronger LTC precautionary saving motive or a stronger bequest motive. I find a qualitatively similar pattern in the relationship between the households' actual stock share and the estimated preference parameters. The size of the response, however, turns out to be much smaller than the prediction from the model.

One possible reason for this low-powered response of choices to the stated preference is that financial advice does not sufficiently take into account of health-related risks, in particular LTC risk. This risk can take the form either of high probability of needing LTC or, as this paper emphasizes, preference for large spending should LTC be needed. The theoretical findings of this paper imply that portfolio advice should be conditioned on these preferences and risks. This paper, by documenting the heterogeneity in preferences and by showing its implications for portfolios for households in different circumstances, provides a roadmap for improving the financial advice to and financial products for households who need to manage financial assets during retirement while facing multiple risks.

## References

- [1] Ameriks, John, Joseph Briggs, Andrew Caplin, Matthew D. Shapiro, and Christopher Tonetti (2015): “Long-Term Care Utility and Late in Life Saving,” NBER Working Paper, No. 20973.
- [2] Ameriks, John, Joseph Briggs, Andrew Caplin, Matthew D. Shapiro, and Christopher Tonetti (2016): “Late-in-Life Risks and the Under-Insurance Puzzle,” NBER Working Paper, No. 22726.
- [3] Ameriks, John, Andrew Caplin, Steven Laufer, and Stijn van Nieuwerburgh (2011): “The Joy of Giving or Assisted Living? Using Strategic Surveys to Separate Public Care Aversion from Bequest Motives,” *Journal of Finance*, 66, 519-561.
- [4] Ameriks, John, Andrew Caplin, Minjoon Lee, Matthew D. Shapiro, and Christopher Tonetti (2015): “The Wealth of Wealthholders,” NBER Working Paper, No. 20972.
- [5] Barsky, Robert B., F. Thomas Juster, Miles S. Kimball, and Matthew D. Shapiro (1997): “Preference Parameters and Behavioral Heterogeneity: An Experimental Approach in the Health and Retirement Study,” *Quarterly Journal of Economics*, 112, 537-579.
- [6] Benzoni, Luca, Pierre Collin-Dufresne, and Robert S. Goldstein (2007): “Portfolio Choice over the Life-Cycle when the Stock and Labor Markets are Cointegrated,” *Journal of Finance*, 62, 2123-2167.
- [7] Berkowitz, Michael K. and Jiaping Qiu (2006): “A Further Look at Household Portfolio Choice and Health Status,” *Journal of Banking and Finance*, 30, 1201-1217.

- [8] Berndt, Ernst R., Bronwyn H. Hall, Robert E. Hall, and Jerry A. Hausman (1974): “Estimation and Inferences in Nonlinear Structural Models,” *Annals of Economic and Social Measurement*, 3, 653-665.
- [9] Bodie, Zvi, Robert C. Merton, and William F. Samuelson (1992): “Labor Supply Flexibility and Portfolio Choice in a Life Cycle Model,” *Journal of Economic Dynamics and Control*, 16, 427-449.
- [10] Brunnermeier, Markus K. and Stefan Nagel (2008): “Do Wealth Fluctuations Generate Time-Varying Risk Aversion? Micro-Evidence on Individuals Asset Allocation,” *American Economic Review*, 98, 713-736.
- [11] Campbell, John Y. (2006): “Household Finance,” *Journal of Finance*, 61, 1553-1604.
- [12] Campbell, John Y. and John H. Cochrane (1999): “By Force of Habit: A Consumption-Based Explanation of Aggregate Stock Market Behavior,” *Journal of Political Economy*, 107, 205-251.
- [13] Cocco, Joao F., Francisco J. Gomes, and Pascal J. Maenhout (2005): “Consumption and Portfolio Choice over the Life Cycle,” *Review of Financial Studies*, 18, 491-533.
- [14] De Nardi, Mariacristina, Eric French, and John B. Jones (2010): “Why Do the Elderly Save? The Role of Medical Expenses,” *Journal of Political Economy*, 118, 39-75.
- [15] De Nardi, Mariacristina, Eric French, and John B. Jones (2013): “Medicaid Insurance in Old Age,” NBER Working Paper, 19151.
- [16] Ding, Jie, Geoffrey Kingston, and Sachi Purcal (2014): “Dynamic Asset Allocation when Bequests are Luxury Goods,” *Journal of Economic Dynamics and Control*, 38, 65-71.
- [17] Fan, Elliott and Ruoyun Zhao (2009): “Health Status and Portfolio Choice: Causality or Heterogeneity?” *Journal of Banking and Finance*, 33, 1079-1088.
- [18] Finkelstein, Amy, Erzo F. P. Luttmer, and Matthew J. Notowidigdo (2009): “Approaches to Estimating the Health State Dependence of the Utility Function,” *American Economic Review*, 99, 116-121.
- [19] Goldman, Dana and Nicole Maestas (2013): “Medical Expenditure Risk and Household Portfolio Choice,” *Journal of Applied Econometrics*, 28, 527-550.

- [20] Gomes, Francisco and Alexander Michaelides (2003): “Portfolio Choice with Internal Habit Formation: A Life-cycle Model with Uninsurable Labor Income Risk,” *Review of Economic Dynamics*, 6, 729-766.
- [21] Huang, Huaxiong and Moshe A. Milevsky (2008): “Portfolio Choice and Mortality-contingent Claims: The General HARA Case,” *Journal of Banking and Finance*, 32, 2444-2452.
- [22] Hurd, Michael D. (2002): “Portfolio Holdings of the Elderly,” In: Luigi Guiso, Michael Haliassos and Tullio Jappelli (eds.), *Households Portfolios*, Cambridge: MIT Press, 27-54.
- [23] Kimball, Miles S., Claudia R. Sahm, and Matthew D. Shapiro (2008): “Imputing Risk Tolerance from Survey Responses,” *Journal of the American Statistical Association*, 103, 1028-1038.
- [24] Klibanoff, Peter, Massimo Marinacci, and Sujoy Mukerji (2005): “A Smooth Model of Decision Making Under Ambiguity,” *Econometrica*, 73, 1849-1892.
- [25] Lockwood, Lee (2014): “Bequest Motives and the Choice to Self-Insure Late-Life Risks,” working paper.
- [26] Love, David A. and Paul A. Smith (2010): “Does Health Affect Portfolio Choice?,” *Health Economics*, 19, 1441-1460.
- [27] Manski, Charles F. and Francesca Molinari (2010): “Rounding Probabilistic Expectations in Survey,” *Journal of Business and Economic Statistics*, 28, 219-231.
- [28] Merton, Robert C. (1971): “Optimum Consumption and Portfolio Rules in a Continuous-Time Model,” *Journal of Economic Theory*, 3, 373-413.
- [29] Pang, Gaobo and Mark Warshawsky (2010): “Optimizing the Equity-Bond-Annuity Portfolio in Retirement: The Impact of Uncertain Health Expenses,” *Insurance: Mathematics and Economics*, 46, 198-209.
- [30] Polkovnichenko, Valery (2007): “Life-cycle Portfolio Choice with Additive Habit Formation Preferences and Uninsurable Labor Income Risk,” *Review of Financial Studies*, 20, 83-124.
- [31] Poterba, James, Steven Venti, and David Wise (2011): “The Composition and Drawdown of Wealth in Retirement,” *Journal of Economic Perspectives*, 25, 95-118.

- [32] Reichling, Felix and Kent Smetters (2015): “Optimal Annuitization with Stochastic Mortality and Correlated Medical Costs,” *American Economic Review*, 105, 3273-3320.
- [33] Rosen, Harvey S. and Stephen Wu (2004): “Portfolio Choice and Health Status,” *Journal of Financial Economics*, 72, 457-484.
- [34] Spaenjers, Christophe and Sven Michael Spira (2014): “Subjective Life Horizon and Portfolio Choice,” HEC Paris Research Paper, No. FI-2013-985.
- [35] Viceira, Luis M. (2001): “Optimal Portfolio Choice for Long-Horizon Investors with Nontradable Labor Income,” *Journal of Finance*, 56, 433-470.
- [36] Vissing-Jorgensen, Annette (2002): “Towards an Explanation of Household Portfolio Choice Heterogeneity: Nonfinancial Income and Participation Cost Structures,” NBER Working Paper 8884.
- [37] Wachter, Jessica A. and Motohiro Yogo (2010): “Why Do Household Portfolio Shares Rise in Wealth?” *Review of Financial Studies*, 23, 3929-3965.

Table 1: Summary Statistics

	Mean	Percentiles				
		10	25	50	75	90
Stock Share	0.548	0.189	0.378	0.553	0.743	0.905
Coupled	0.673					
Male	0.647					
Age	68.0	58	62	67	73	78
Employer-sponsored sample	0.219					
Health ( $\geq$ Good)	0.948					
Education (Post college)	0.404					
Education (College)	0.330					
Financial Wealth (\$)	1,101,468	153,000	344,000	723,665	1,356,211	2,399,317
Retired	0.54					
Wage income(\$)	45,145	0	0	0	72,900	140,000
Annuity income (\$) <sup>a</sup>	36,929	0	0	17,784	50,400	81,852
Expected Annuity <sup>a</sup> income (\$)	36,969	0	0	0	48,000	83,736
Home equity	354,204	25,000	120,000	230,000	411,000	735,500
Prob. need LTC <sup>b</sup>	0.430	0.05	0.15	0.45	0.75	0.85
Prob. live approx. 10 more years <sup>c</sup>	0.753	0.45	0.65	0.85	0.95	0.95
Having LTC insurance	0.234					

Notes: The tabulation is conditioned on having responses to all the variables used in this paper (all the SSQs in addition to the variables in this table). N=5,471.

<sup>a</sup> Annuity income is the sum of Social Security income, defined benefit pension income and immediate annuity income, for retired households. It is set to zero for non-retired households. Expected annuity income is the sum of expected values of Social Security income, defined benefit pension income and immediate annuity income, for non-retired households. It is set to zero for retired households.

<sup>b</sup> The subjective probability of being in need of a LTC service at least for 1 year in the remaining life.

<sup>c</sup> The subjective probability of living up to at least age  $\min(\{75, 85, 95\} \cap \{t | t \geq \text{age} + 5\})$ . For example, for a respondent whose age is 75, it is the probability of living up to age 85.

Table 2: Estimated Distributions of the Preference Parameters and Survey Response Errors

(a) Estimated distribution parameters

Parameter	Estimate	S.E.
$\mu_\theta$	-1.322	(0.013)
$\sigma_\theta$	0.691	(0.007)
$\kappa$	-10.82K	(0.43K)
$\mu_{LTC}$	1.292	(0.034)
$\sigma_{LTC}$	2.317	(0.029)
$\mu_{\kappa,LTC}$	-35.97K	(0.64K)
$\sigma_{\kappa,LTC}$	24.67K	(0.29K)
$\mu_{Beq}$	2.487	(0.042)
$\sigma_{Beq}$	3.548	(0.047)
$\mu_{\kappa,Beq}$	38.74K	(1.11K)
$\sigma_{\kappa,Beq}$	45.10K	(0.69K)
$\sigma_{\varepsilon 11}$	0.177	(0.002)
$\sigma_{\varepsilon 12}$	0.109	(0.001)
$\sigma_{\varepsilon 21}$	14.98K	(0.14K)
$\sigma_{\varepsilon 22}$	11.98K	(0.15K)
$\sigma_{\varepsilon 23}$	8.11K	(0.09K)
$\sigma_{\varepsilon 31}$	15.24K	(0.14K)
$\sigma_{\varepsilon 32}$	9.84K	(0.17K)
$\sigma_{\varepsilon 33}$	19.33K	(0.25K)
N	5,471	
Log-likelihood	-125,903	

(b) Implied distributions of preference parameters

Parameter	Percentiles					Mean
	10	25	50	75	90	
$\theta_i$	0.11	0.17	0.27	0.43	0.65	0.34
$\zeta_{LTC,i}$	0.19	0.76	3.64	17.37	70.91	53.32
$\zeta_{Beq,i}$	0.12	1.10	12.03	131.65	1134.69	6510.37
$\kappa_{LTC}$	-67.59K	-52.61K	-35.97K	-19.33K	-4.35K	-35.97K
$\kappa_{Beq}$	-19.06K	8.32K	38.74K	69.16K	96.54K	38.74K

Table 3: Stock Share Regression: Using Raw Responses to SSQs

	1	2
SSQ1 (10-20%)	0.028*** (0.010)	0.017* (0.010)
SSQ1 (20-33%)	0.048*** (0.010)	0.034*** (0.010)
SSQ1 (33-50%)	0.059*** (0.012)	0.047*** (0.012)
SSQ1 (50-75%)	0.081*** (0.013)	0.070*** (0.013)
SSQ1 (75-100%)	0.037 (0.026)	0.035 (0.026)
SSQ2 (Share of wealth for LTC)	-0.048** (0.020)	-0.034* (0.020)
SSQ3 (Share of wealth for bequest)	0.008 (0.015)	0.013 (0.015)
Coupled		-0.014* (0.009)
Male		0.026*** (0.008)
Age		0.001 (0.001)
Employer-sponsored		-0.050*** (0.010)
Health (Good)		-0.022 (0.016)
Post college degree		0.036** (0.015)
College degree		0.028** (0.014)
Log wealth		0.018*** (0.004)
Retired		0.068 (0.095)
Log wage		0.009** (0.004)
Log annuity income		-0.008 (0.007)
Log expected annuity income		-0.008 (0.006)
LTC prob.		-0.024** (0.012)
Longevity prob.		0.045*** (0.017)
LTCI		-0.011 (0.008)
Log home equity		0.002* (0.001)
N	5471	5471
R <sup>2</sup>	0.011	0.034

Note: For SSQ1, the most risk averse category (i.e., willing to risk 0-10% of their income to have a 50% chance of doubling income) is the omitted category. For SSQ2 the raw response is defined as the share of wealth the respondent allots for the LTC state, averaged over the three questions. For SSQ3, it is the share of wealth bequeathed, averaged over the three questions. For a description of the controls, see the note to Table 1.

\*= p<0.1, \*\*=p<0.05, \*\*\*=p<0.01



Table 4: Stock Share Regression: Using Estimated Preference Parameters (Cardinal Proxies)

	1	2
$\log\theta$	0.027*** (0.006)	0.023*** (0.006)
$\log\zeta_{LTC}$	-0.010*** (0.002)	-0.009*** (0.002)
$\log\zeta_{Beq}$	0.002 (0.002)	0.002 (0.001)
$\kappa_{LTC}$ (in \$10K)	-0.003 (0.002)	-0.004* (0.002)
$\kappa_{Beq}$ (in \$10K)	0.003* (0.002)	0.001 (0.002)
Coupled		-0.012 (0.009)
Male		0.016* (0.009)
Age		0.001 (0.001)
Employer-sponsored		-0.043*** (0.010)
Health (Good)		-0.012 (0.017)
Post college degree		0.023 (0.015)
College degree		0.022* (0.014)
Log wealth		0.018*** (0.004)
Retired		0.043 (0.096)
Log wage		0.010** (0.004)
Log annuity income		-0.005 (0.007)
Log expected		-0.009 (0.006)
Log annuity income		-0.033** (0.012)
LTC prob.		-0.033** (0.012)
Longevity prob.		0.038** (0.017)
LTCI		-0.010 (0.008)
Log home equity		0.002 (0.001)
N	5471	5471
$R^2$	0.009	0.032

Note: See Section 5 for construction of the cardinal proxies. For a description of the controls, see the note to Table 1. Standard errors are bootstrapped, with 100 repetitions.

\*=  $p < 0.1$ , \*\*= $p < 0.05$ , \*\*\*= $p < 0.01$

Table 5: Implied Changes in Stock Share by a Two-standard-deviation Increase in Each Preference Parameters

	1 (without control)	2 (with control)
$\log\theta$	0.037	0.031
$\log\zeta_{LTC}$	-0.046	-0.040
$\log\zeta_{Beq}$	0.012	0.011
$\kappa_{LTC}$	-0.017	-0.021
$\kappa_{Beq}$	0.026	0.010

Table 6: Investigation of the effect of home equity

	1	2	3
$\log\theta$	0.023*** (0.006)	0.011** (0.005)	0.017*** (0.005)
$\log\zeta_{LTC}$	-0.009*** (0.002)	-0.007*** (0.002)	-0.006*** (0.002)
$\log\zeta_{Beq}$	0.002 (0.001)	0.001 (0.001)	0.001 (0.001)
$\kappa_{LTC}$ (in \$10K)	-0.004* (0.002)	-0.003 (0.002)	-0.003 (0.002)
$\kappa_{Beq}$ (in \$10K)	0.001 (0.002)	0.001 (0.001)	0.001 (0.001)
Control	Y	Y	Y
N	5471	5471	5471
$R^2$	0.032	0.011	0.038

Note: Specifications are defined as below:

1-Baseline

2-Home equity included as a safe asset in the left hand side variable.

3-Home equity included as a risky asset in the left hand side variable.

Standard errors are bootstrapped, with 100 repetitions.

\*= p<0.1, \*\*=p<0.05, \*\*\*=p<0.01

Table 7: Calibration of Parameters for Baseline Model

(a) Heterogeneous preference parameters

Parameter		Value
$\theta$	High	0.53
	Medium	0.27
	Low	0.13
$\zeta_{LTC}$	High	36.93
	Medium	3.64
	Low	0.36
$\zeta_{Beq}$	High	418.80
	Medium	12.03
	Low	0.34
$\kappa_{LTC}$	High	-11.30K
	Medium	-35.97K
	Low	-60.64K
$\kappa_{Beq}$	High	83.84K
	Medium	38.74K
	Low	-6.36K

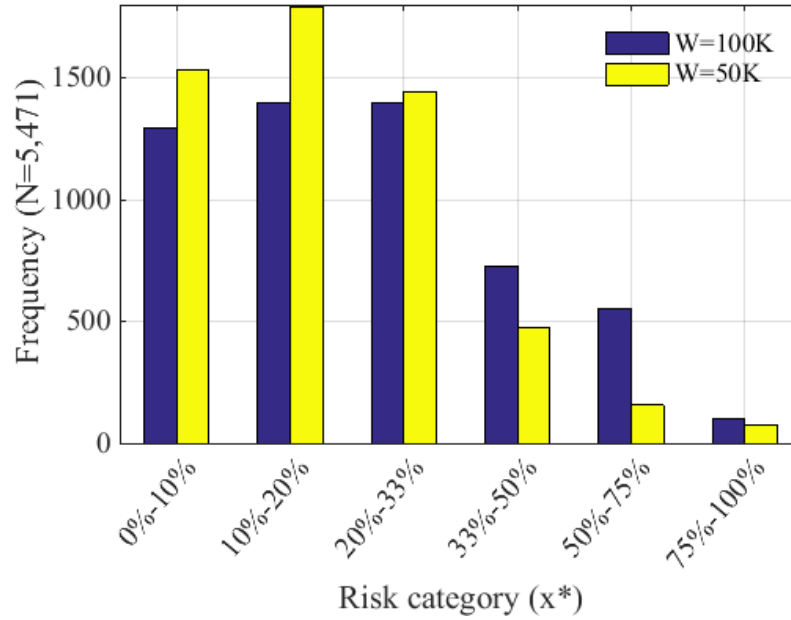
Note: For each parameter, medium value is the mean value of the distribution (exponential of mean if the distribution of the parameter is log-normal) while high (low) value is the mean plus (minus) one standard deviation (again exponential of those values if the distribution of the parameters is log-normal).

(b) Other parameters

Parameters	Value	Target/Source
$\kappa$	-10.82K	VRI estimation
$\beta$	0.96	Standard
$PC$	$-\kappa_{LTC} + 10K$	Ameriks et al. (2015b)
$\pi_{ss'}$		HRS estimation
$\bar{R}_f$	1.02	Cocco et al. (2005)
$\sigma_\eta$	0.17	Cocco et al. (2005)
$\mu_s$	0.025	VRI stock share level
$\bar{y}$	$\{\$45K, \$90K, \$120K\}$	VRI data
$\lambda$	0.5	VRI data
$\sigma_{nu}^2$	0.07	Cocco et al. (2005)

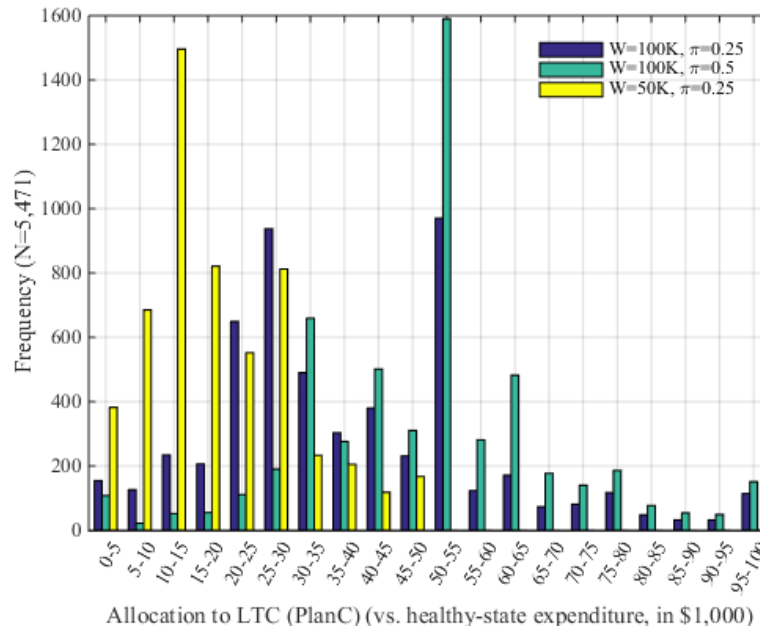
Figure 1: Distribution of Responses to SSQs

(a) SSQ1



Note: Risk categories show the downside risk that is accepted for a 50 percent chance of doubling income. 0-10% is the most risk averse group while 75-100% is the most risk tolerant one.

(b) SSQ2



(c) SSQ3

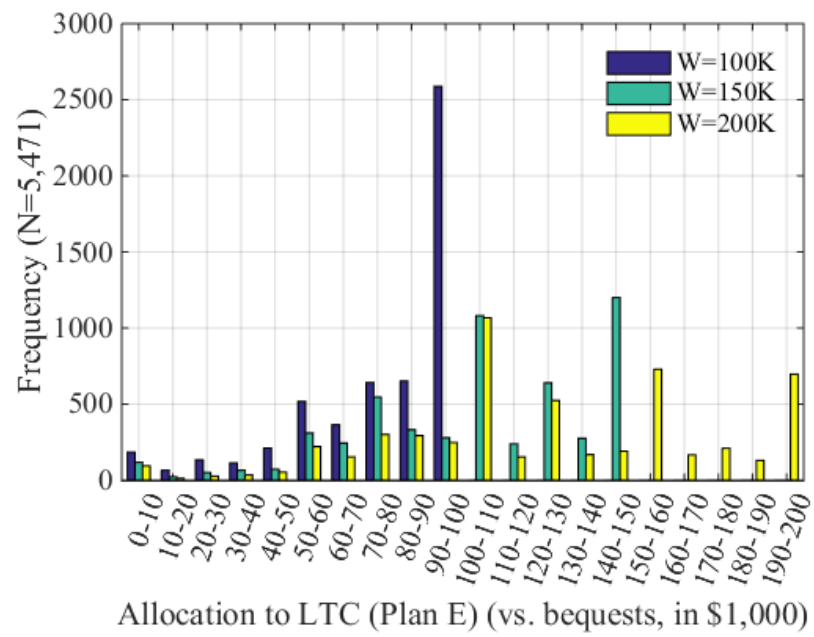
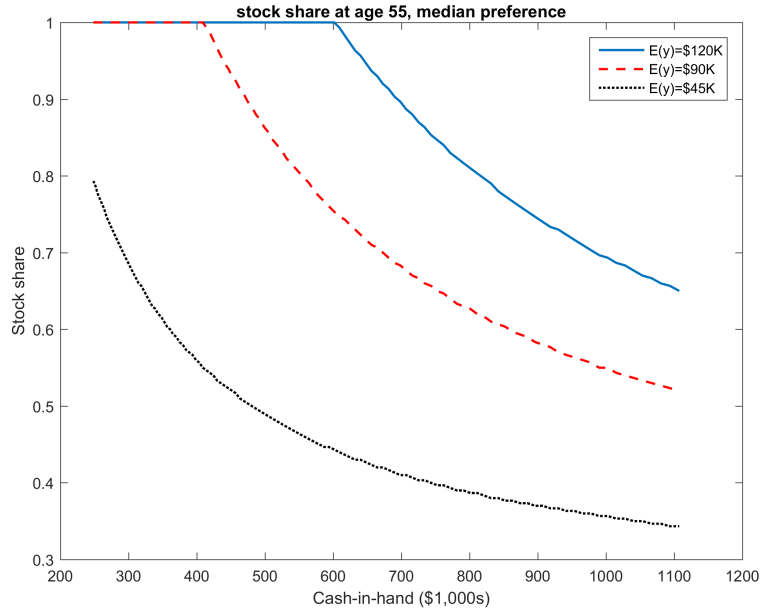
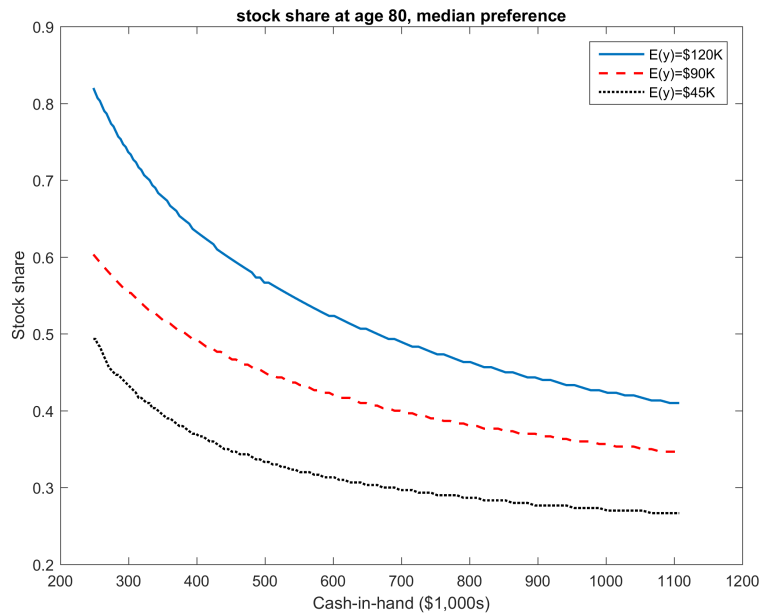


Figure 2: Stock Share Policy Functions (under the median preference parameters)

(a) Age 55



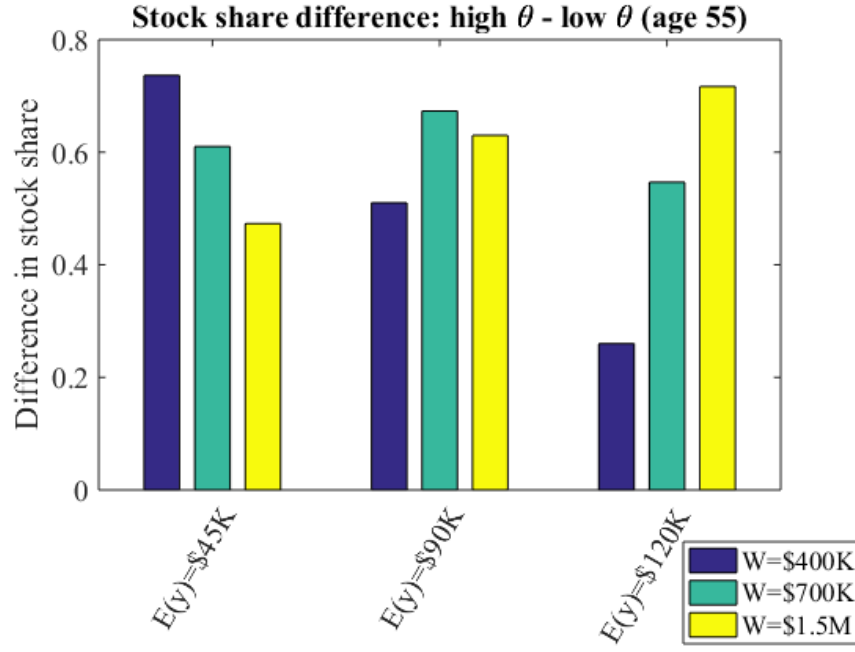
(b) Age 80



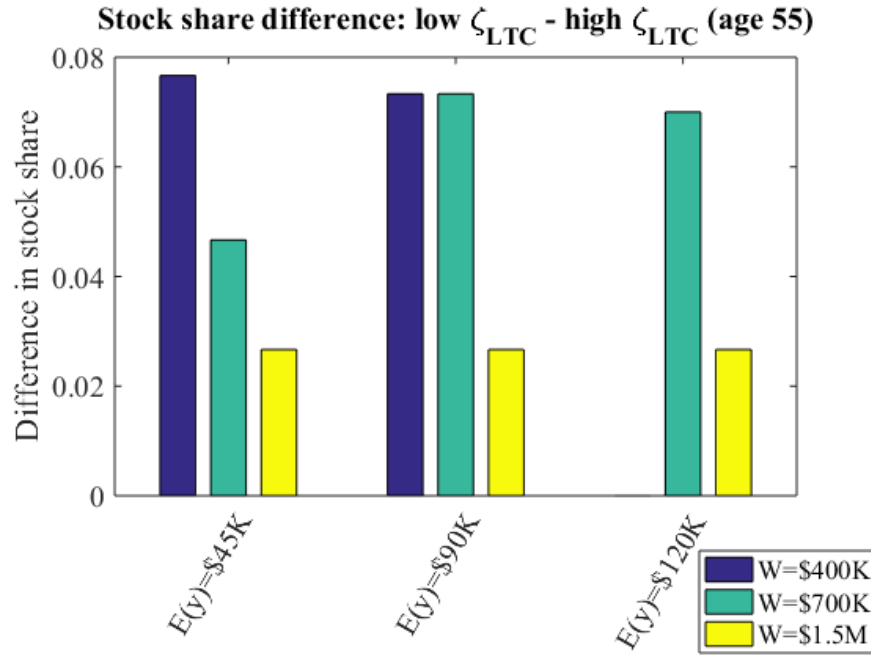
Note: Figure shows the optimal stock share policy function for healthy males, under various values of average income and median preferences. The horizontal axis is financial wealth at the beginning of the period (in \$1,000s), and the vertical axis is the optimal stock share.

Figure 3: Effects of Heterogeneous Preference Parameters on Optimal Stock Share (age 55, healthy, male)

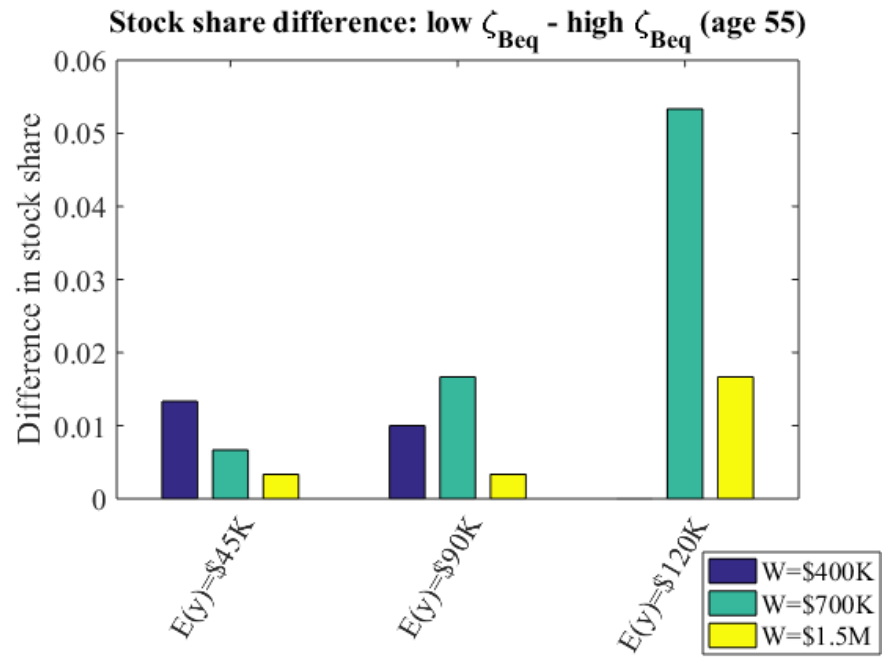
(a) Effect of  $\theta$



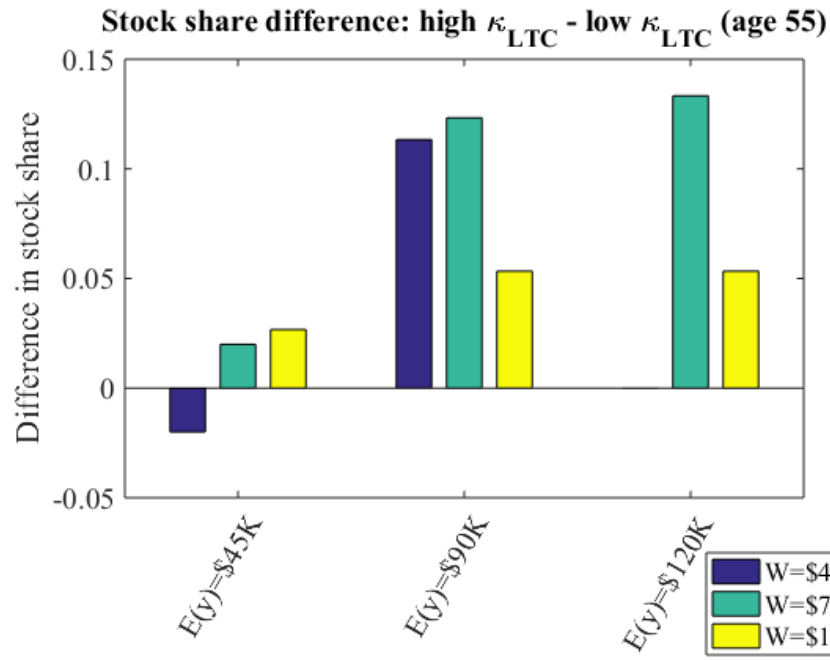
(b) Effect of  $\zeta_{LTC}$



(c) Effect of  $\zeta_{Beq}$

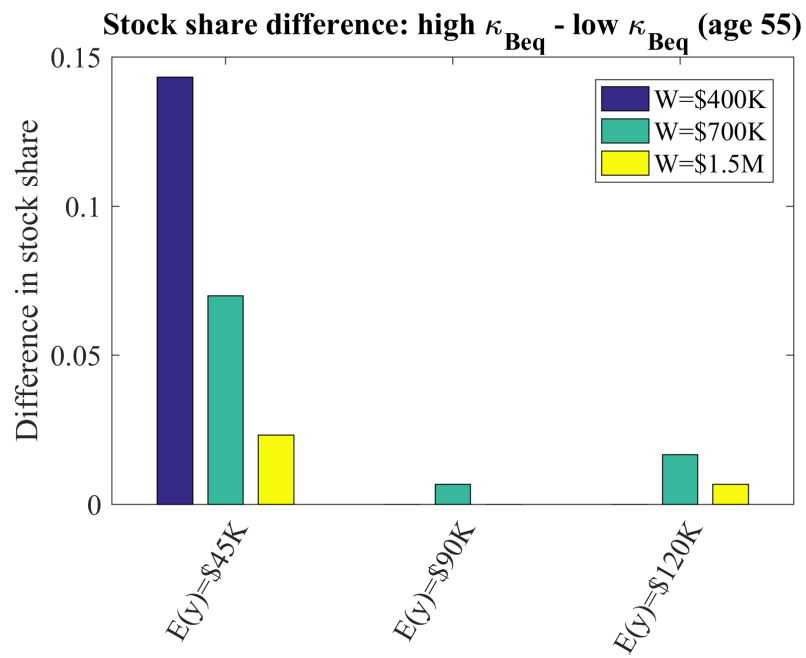


(d) Effect of  $\kappa_{LTC}$





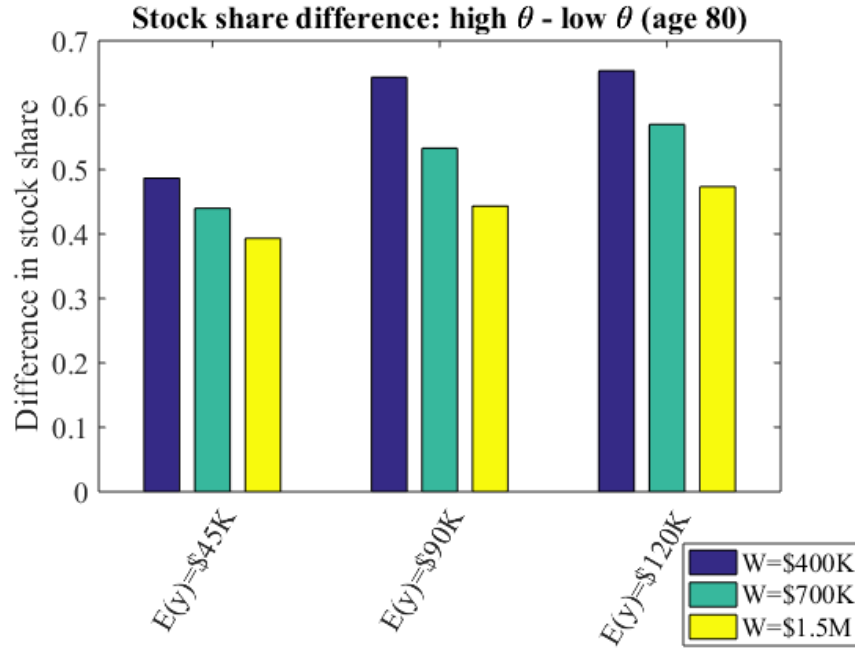
(e) Effect of  $\kappa_{Beq}$



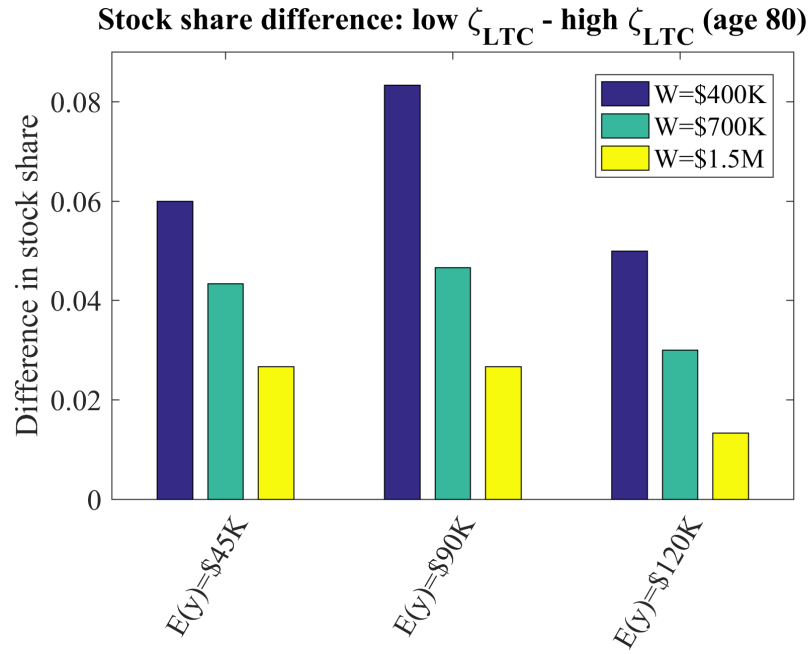
Note: Figure shows the difference in the optimal stock share across different values for preference parameters, under various wealth and mean income levels, at age 55 for healthy males.

Figure 4: Effects of Heterogeneous Preference Parameters on Optimal Stock Share (age 80, healthy, male)

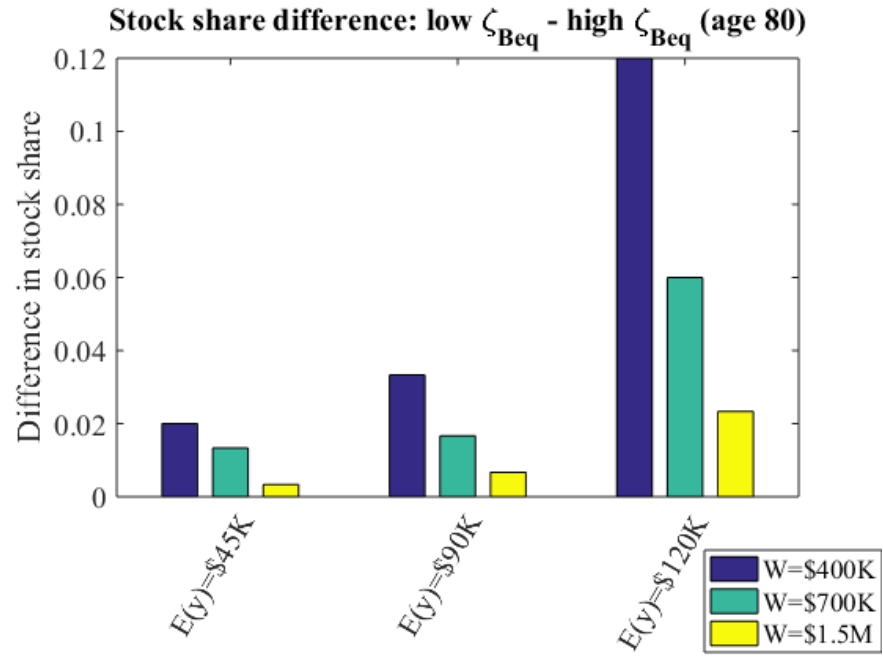
(a) Effect of  $\theta$



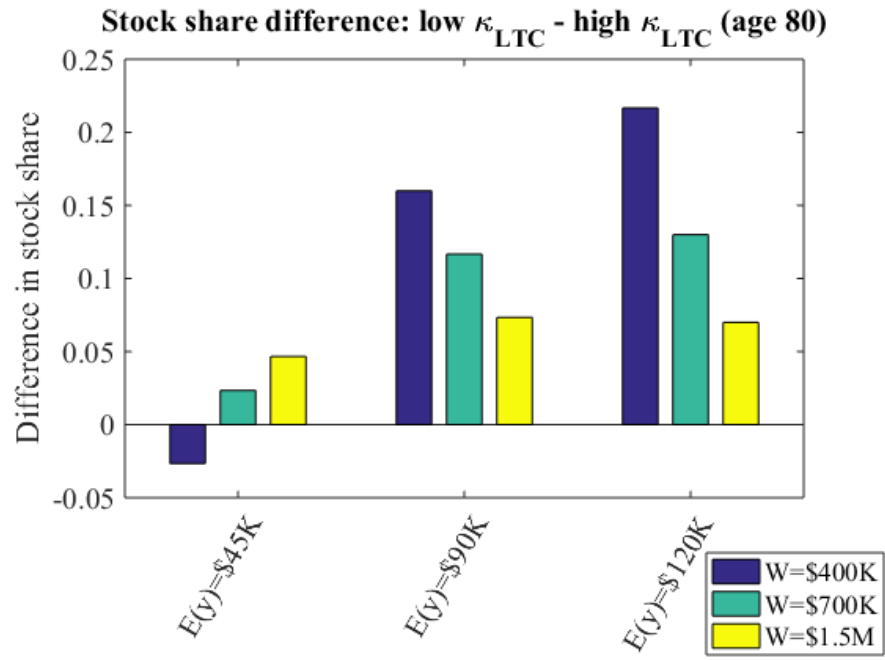
(b) Effect of  $\zeta_{LTC}$



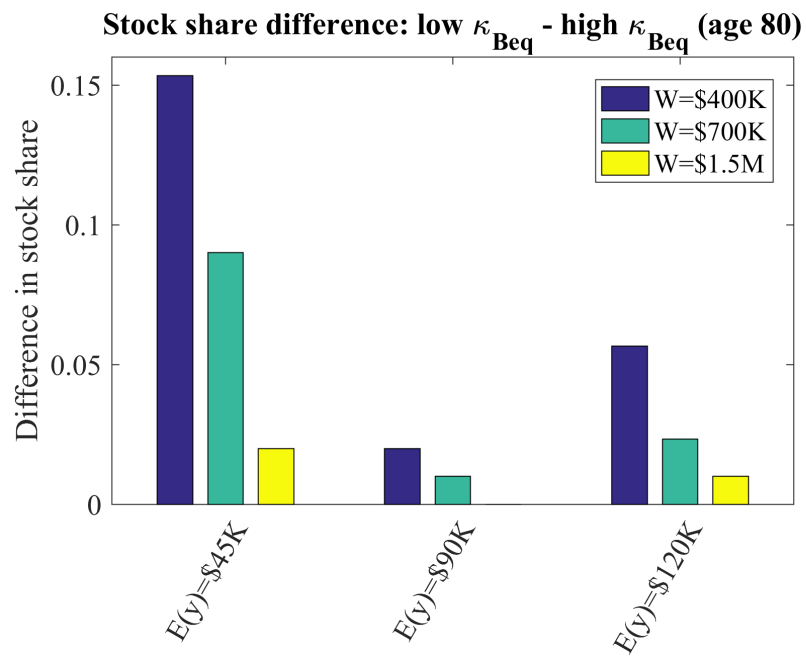
(c) Effect of  $\zeta_{Beq}$



(d) Effect of  $\kappa_{LTC}$



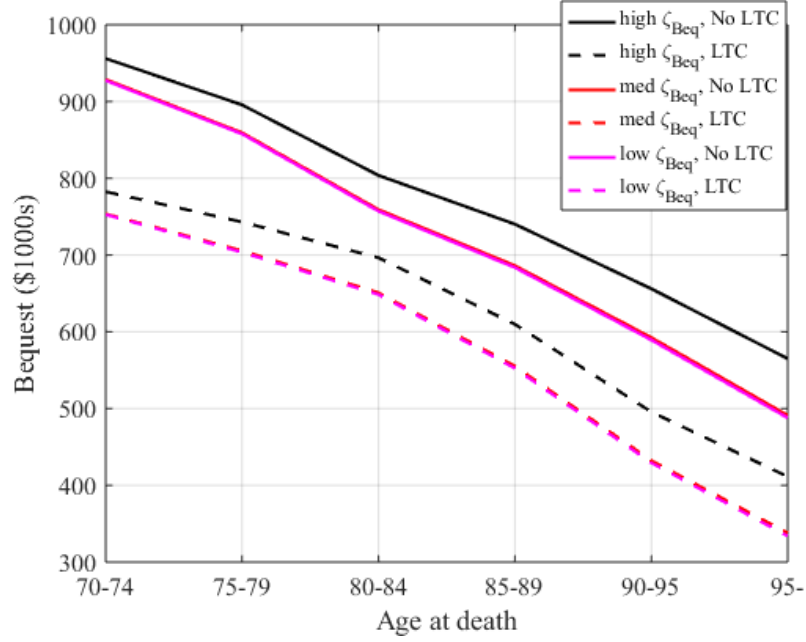
(e) Effect of  $\kappa_{Beq}$



Note: Figure shows the difference in the optimal stock share across different values for preference parameters, under various wealth and mean income levels, at age 80 for healthy males.

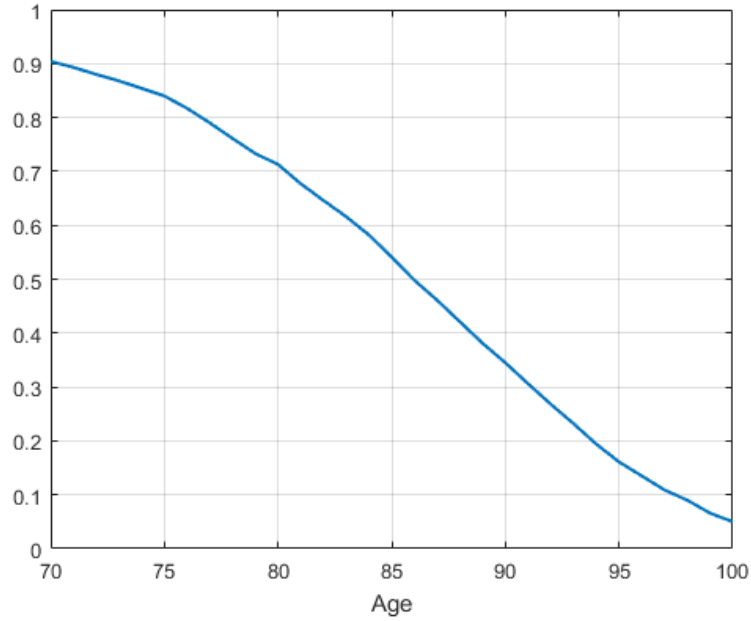
Figure 5: Mechanism Behind the Effect of  $\zeta_{Beq}$

(a) Effect of LTC shock on bequests

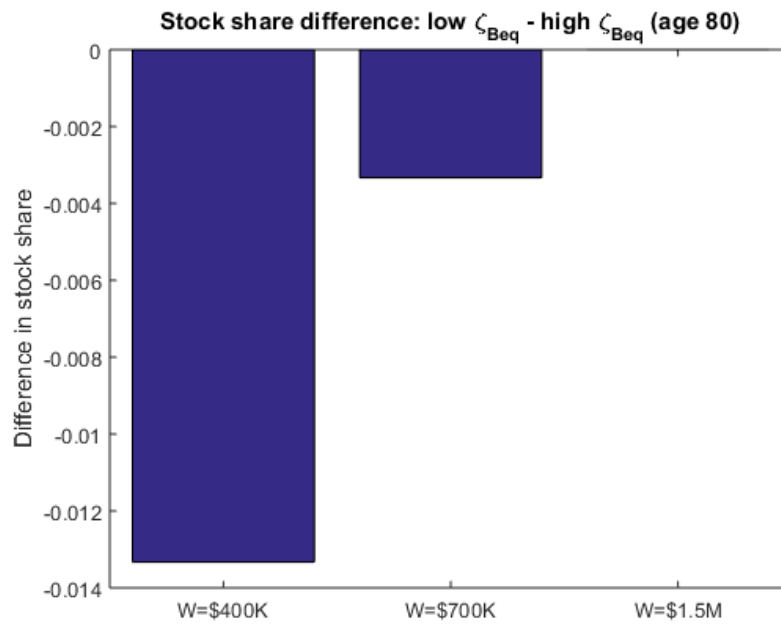


Note: Figure shows the average amount of bequest conditional on age at death,  $\zeta_{Beq}$ , and whether the household ever had LTC shock during its lifetime. Averages are calculated from 10,000 simulations for each  $\zeta_{Beq}$  value. Each simulation starts with wealth of \$700,000 and  $\bar{y} = \$90,000$ .

(b) Survival rate



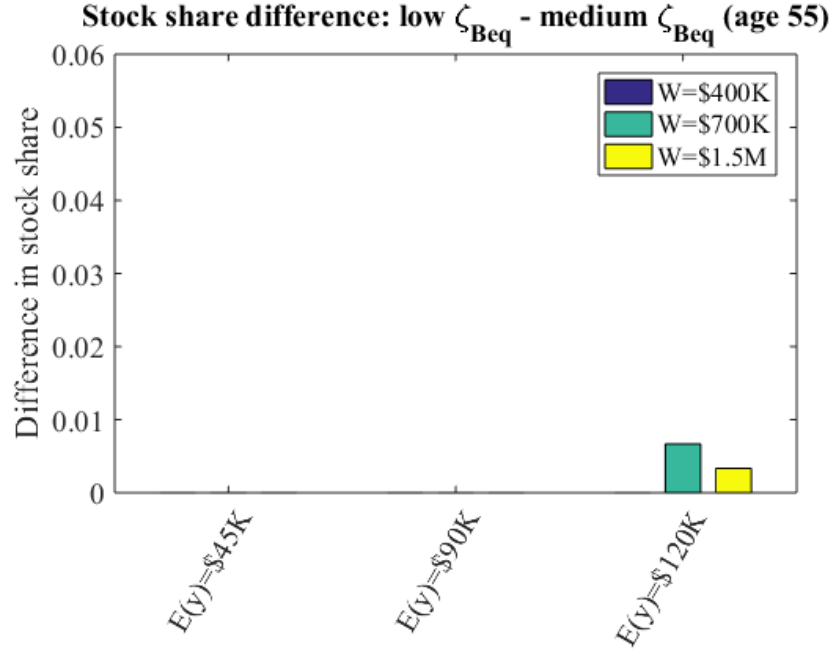
(c) Effect of  $\zeta_{Beq}$  under no health-related risks



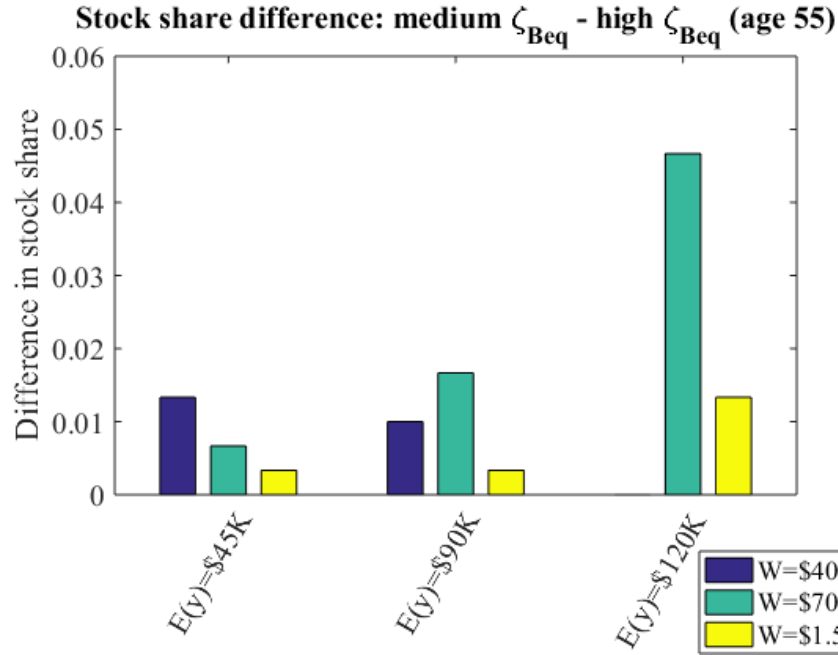
Note: Figure shows the difference in the optimal stock share across different values of  $\zeta_{Beq}$  under no LTC risk and mortality risk. A household lives up to age 110 and then dies with probability one. The figure is drawn for healthy males at age 55, with  $\bar{y} = \$90,000$ , under various wealth levels.

Figure 6: Effect of one-standard-deviation difference in  $\zeta_{Beq}$

(a) Limited effects in the lower range of  $\zeta_{Beq}$



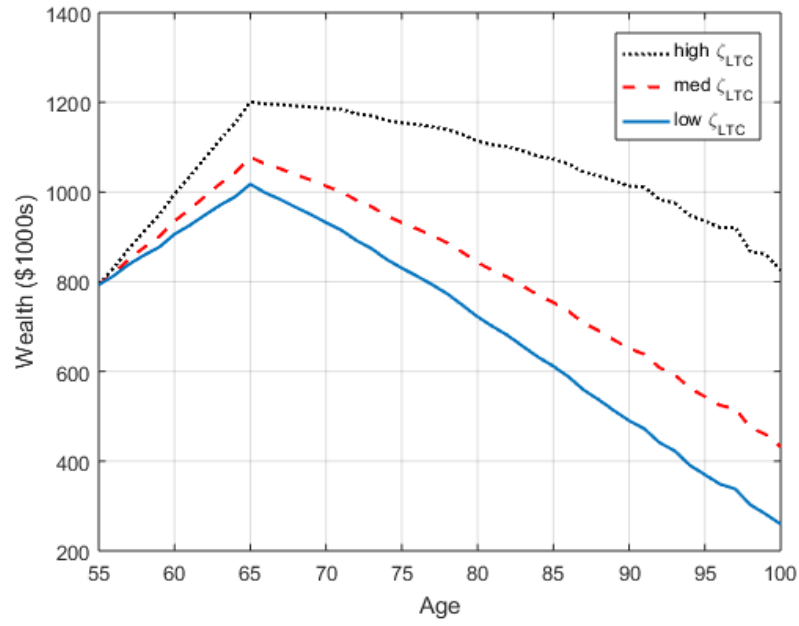
(b) Larger effects in the higher range of  $\zeta_{Beq}$



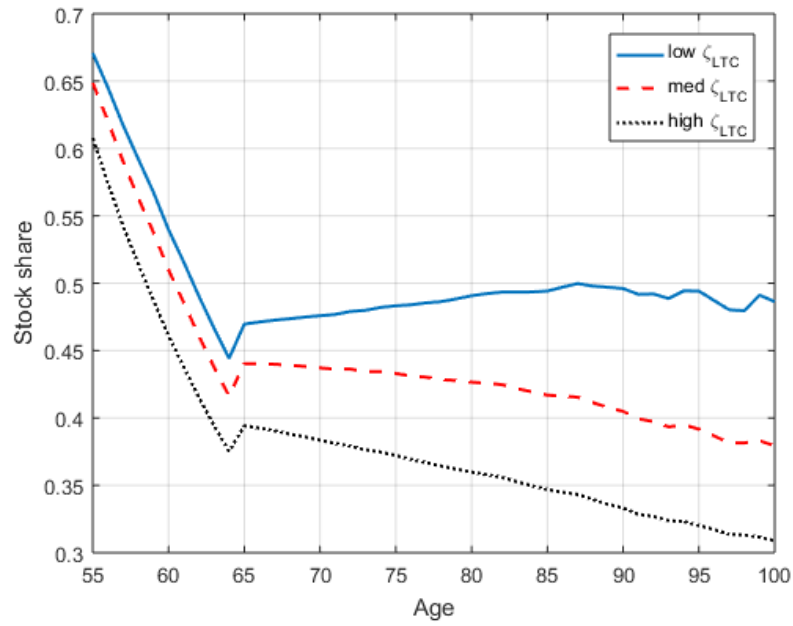
Note: Figure is constructed in the same way as in Figure 5(c), but in this Figure I calculate the effect of one-standard-deviation differences in  $\zeta_{Beq}$ .

Figure 7: Life-cycle profiles for wealth and stock share under various  $\zeta_{LTC}$  and  $\zeta_{Beq}$

(a) Wealth with various  $\zeta_{LTC}$

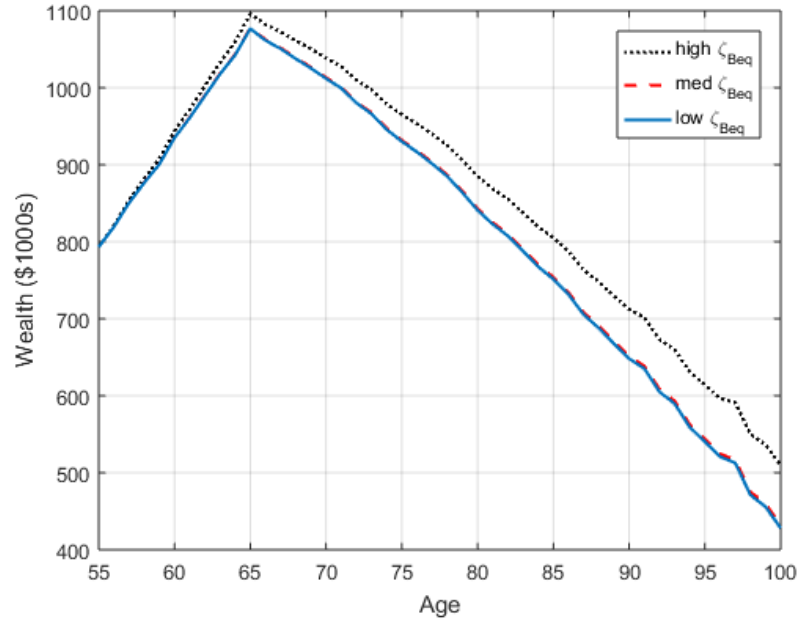


(b) Stock share with various  $\zeta_{LTC}$

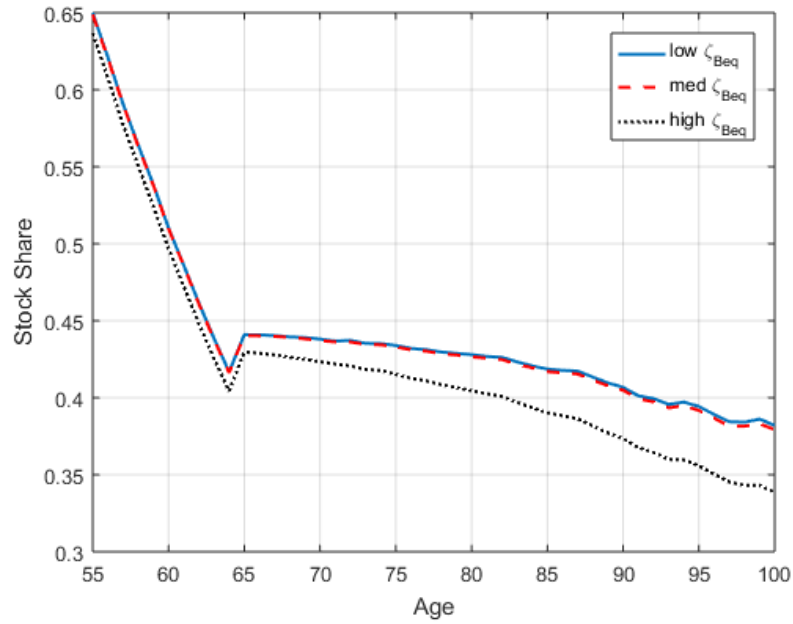




(c) Wealth with various  $\zeta_{Beq}$



(d) Stock share with various  $\zeta_{Beq}$



Note: Figure shows the life-cycle profiles of wealth and stock share under various values of  $\zeta_{LTC}$  and  $\zeta_{Beq}$ . Profiles are calculated from 10,000 simulations for each parameter value. Each simulation starts with wealth of \$700,000 and  $\bar{y} = \$90,000$ .

## A Appendix: Details on Strategic Survey Questions (SSQs)

Table A1 shows the exact wordings and parameter values used for each type of SSQ. Each type is asked multiple times with different amounts of given resources ( $W$ ) and/or different likelihoods of relevant events ( $\pi$ ).

Figure A1 shows an example of the interface—a bar with a slider—that is used in SSQ2 and SSQ3 to help respondents understand the underlying trade-off in allocating their resources. In this figure, a respondent is answering the first question of SSQ2 (allocating \$100,000 between Plan C and Plan D, where the chance of needing a LTC service in the next year is 25%). When the respondent first sees this screen, it does not have a slider and there is only an empty bar. Once the respondent clicks on the bar, the slider appears where she clicked. The purpose of this design is to prevent any anchoring effect of an arbitrarily-chosen initial location. After the initial click, the respondent can move the slider to the left and right to adjust the allocation. Whenever the slider is moved, the numbers below the bar, which show the amount of resources available for each state under the current allocation, automatically update. In this way, the respondent can see the consequence of her decision without needing to make complex calculations.

Table A1: Strategic Survey Questions

(a) SSQ1 (Risk tolerance)

Set up	<p>Suppose you are 80 years old. Suppose, further, that for the next year:</p> <ul style="list-style-type: none"> <li>• You live alone, rent your home, and pay all your own bills.</li> <li>• You are in good health and will remain in good health.</li> <li>• You will have no medical bills or other unexpected expenses.</li> <li>• You do not work.</li> </ul>
Hypothetical financial products	<ul style="list-style-type: none"> <li>• Plan A guarantees that you will have <math>\\$W</math> for spending next year.</li> <li>• Plan B will possibly provide you with more money, but is less certain. There is a 50% chance that Plan B would double your money, leaving you with <math>\\$2W</math>, and a 50% chance that it would cut it by <math>x\%</math>, leaving you with <math>\$(1 - 0.01 \times x)W\$.</math></li> </ul>
Rules	<ul style="list-style-type: none"> <li>• You have no other assets or income, and so the only money you have available for all your spending next year is from either Plan A or Plan B.</li> <li>• Any money that is not spent at the end of next year cannot be saved for the future.</li> <li>• You cannot give any money away or leave it as a bequest.</li> <li>• If you need anything next year, you have to pay for it. No one else can buy anything for you.</li> <li>• At the end of next year you will be offered the same choice with another <math>\\$W</math> for following year.</li> </ul>
Parameters asked	$W = \$100,000$ and $\$50,000$ .

(b) LTC-state utility function

Set up	<ul style="list-style-type: none"> <li>• You are 80 years old, live alone, rent your home, and pay all your own bills.</li> <li>• There is a <math>\pi</math> chance that you will need help with ADLs for all of next year.</li> <li>• There is a <math>(1 - \pi)</math> chance that you will not need any help at all with ADLs for all of next year.</li> <li>• You have <math>\\$W</math> to divide between two plans for the next year.</li> <li>• At the end of next year you will be offered the same choice with another <math>\\$W</math> for following year.</li> </ul>
Hypothetical financial products	<ul style="list-style-type: none"> <li>• Plan C is hypothetical ADL insurance that gives you <math>\$(1/\pi)\$</math> for each dollar invested if you do need help with ADLs.</li> <li>• Plan D gives you <math>\\$1</math> for each dollar invested only if you do not need help with ADLs.</li> </ul>
Rules	<ul style="list-style-type: none"> <li>• You can only spend money from Plan C or Plan D next year. You do not have any other money.</li> <li>• Any money that is not spent at the end of next year cannot be saved for the future, given away, or left as a bequest.</li> <li>• Regardless of whether or not you need help with ADLs, your hospital, doctor bills, and medications are completely paid by insurance.</li> <li>• Other than Plan C, you have no other resources available to help with your long-term care. You have to pay for any long-term care you may need from Plan C.</li> <li>• There is no public-care option or Medicaid if you do not have enough money to pay for a nursing home or other long-term care.</li> <li>• An impartial third party that you trust will verify whether or not you need help with ADLs immediately, impartially, and with complete accuracy.</li> </ul>
Parameters asked	$(W, \pi) = (\$100,000, 25\%), (\$100,000, 50\%),$ and $(\$50,000, 25\%).$

(c) SSQ3 (Bequest utility function)

Set up	<ul style="list-style-type: none"><li>• You are 85 years old, live alone, rent your home, and pay all your own bills.</li><li>• You know with certainty that you will live for only 12 more months and that you will need help with ADLs for the entire 12 months.</li><li>• You have <math>\\$W</math> to split into the following two plans.</li></ul>
Hypothetical financial products	<ul style="list-style-type: none"><li>• Plan E is reserved for your spending. From Plan E, you will need to pay all of your expenses, including long-term care and any other wants, needs, and discretionary purchases.</li><li>• Plan F is an irrevocable bequest.</li></ul>
Rules	<ul style="list-style-type: none"><li>• You have no money other than <math>\\$W</math>.</li><li>• No one—including friends or family—can take care of you for free. Long-term care must be purchased at market rates.</li><li>• Any money in Plan E that you do not spend cannot be given away or left as a bequest.</li><li>• Bequests from Plan F are not subject to any taxation.</li><li>• You have full insurance that covers all of your hospital, doctor, and medications, but you have no long-term care insurance.</li><li>• There is no public-care option or Medicaid if you do not have enough money to pay for a nursing home or other long-term care.</li></ul>
Parameters asked	$W = \$100,000, \$150,000, \text{ and } \$200,000.$

Figure A1: Example of the SSQ Interface

Section 1

Section 2

Section 3

Section 4

[Click here for complete scenario](#)

Please make your decision on splitting money into Plan C and Plan D by clicking on the scale below. To put more money in Plan C, move the slider to the left. To put more money in Plan D, move the slider to the right. The numbers in the boxes will change as you move the slider to let you know how much you will receive if you need long term care and if you do not.

Please move the slider to see how it works. When you are ready, place the slider at the split you want and click NEXT to enter your choice.

Plan C

\$100,000

You will have the above amount if you need help with ADLs.

Plan D

\$75,000

You will have the above amount if you do not need help with ADLs.

## **B Appendix: Estimation of Preference Parameter Distribution Conditional on Covariates**

Table B1 shows the results from the estimation conditional on the covariates. For most of the covariates, they have offsetting effects on the multiplier and the necessity parameter so their effect on each saving motivation is ambiguous. For example, for more educated respondents, the utility multiplier for the LTC state tends to be smaller while the corresponding necessity parameter tends to be more negative (i.e., the minimum expenditure in the LTC state increases).

The following variables have unambiguous effects on the saving motives. Older respondents tend to be more risk averse. They also have a stronger precautionary saving motive for LTC and a stronger bequest motive. The strength of the bequest motive is also associated with a more pessimistic expectation regarding their own health (both LTC and longevity expectations) and a lower income level.

Table B1: Estimated Distributions of the Preference Parameters and Survey Response Errors, Conditional on Covariates

	Preference Parameters					
	$\mu_\theta$	$\kappa$	$\mu_{LTC}$	$\mu_{\kappa,LTC}$	$\mu_{Beq}$	$\mu_{\kappa,Beq}$
Constant	-1.186*** (0.290)	-6.304 (9.814)	2.000* (0.848)	-22.961 (14.554)	3.730*** (0.966)	11.367 (20.753)
Coupled	-0.054* (0.025)	3.767*** (0.872)	0.214** (0.078)	4.326*** (1.274)	0.639*** (0.092)	2.472 (1.783)
Male	0.251*** (0.023)	-2.340** (0.850)	-0.809*** (0.072)	-10.480*** (1.281)	-0.555*** (0.091)	-17.051*** (1.810)
Age	-0.005** (0.002)	-0.177** (0.066)	0.030*** (0.006)	-0.030 (0.092)	0.030*** (0.007)	-0.193 (0.139)
Employer-sponsored	-0.070* (0.030)	4.447*** (1.105)	0.402*** (0.088)	7.968*** (1.467)	0.217* (0.098)	-5.566 (2.232)
Health ( $\geq$ Good)	-0.237*** (0.051)	9.467*** (1.453)	0.450* (0.166)	14.841*** (2.212)	1.173*** (0.212)	19.757*** (2.928)
Post college degree	0.219*** (0.037)	-8.268*** (1.500)	-0.825*** (0.117)	-14.675*** (2.249)	-0.573*** (0.155)	-7.265* (3.660)
College degree	0.097** (0.034)	-3.701*** (1.454)	-0.634*** (0.105)	-5.481** (2.139)	-0.477*** (0.153)	-4.498 (3.527)
Log wealth	-0.019 (0.010)	-0.272 (0.366)	-0.049 (0.028)	0.304 (0.527)	-0.165*** (0.034)	-4.839*** (0.737)
Retired	0.421 (0.308)	-19.869** (9.931)	0.294 (0.809)	-41.484 ** (16.254)	-0.768 (1.072)	-53.996 ** (21.150)
Log wage	-0.034** (0.010)	0.798*** (0.243)	0.013 (0.040)	0.711 (0.546)	-0.074 (0.047)	0.223 (0.665)
Log annuity income	-0.027 (0.020)	1.060 (0.653)	-0.077* (0.039)	3.957 *** (1.141)	-0.060 (0.073)	2.438* (1.423)
Log expected annuity income	0.046* (0.022)	-1.604** (0.704)	-0.082 (0.058)	-0.623 (1.069)	-0.048 (0.072)	-2.247 (1.529)
LTC prob.	0.043 (0.034)	-1.693 (1.168)	-0.328** (0.109)	-12.941*** (1.713)	0.202 (0.126)	3.059*** (2.385)
Longevity prob.	0.284*** (0.047)	-6.592*** (1.721)	-0.458** (0.153)	-6.877*** (2.492)	-0.669*** (0.173)	-1.576 (3.737)
LTCI	-0.124*** (0.025)	4.133*** (0.938)	-0.156* (0.090)	-0.917* (1.336)	-0.279* (0.104)	2.071 (1.979)
Log home equity	0.011*** (0.003)	-0.485*** (0.118)	-0.042*** (0.009)	-0.763*** (0.185)	-0.014 (0.010)	0.317 (0.227)
Heterogeneity $\sigma$	0.679*** (0.007)	n/a	2.303*** (0.030)	24.523*** (0.408)	3.559*** (0.046)	42.558*** (0.662)
Measurement error						
$\sigma_{\varepsilon 11}$			0.175*** (0.002)			
$\sigma_{\varepsilon 12}$			0.109*** (0.001)			
$\sigma_{\varepsilon 21}$			14.643*** (0.137)			
$\sigma_{\varepsilon 11}$			11.778*** (0.149)			
$\sigma_{\varepsilon 11}$			7.984*** (0.091)			
$\sigma_{\varepsilon 11}$			14.909*** (0.152)			
$\sigma_{\varepsilon 11}$			9.711*** (0.163)			
$\sigma_{\varepsilon 11}$			19.194*** (0.236)			
Log-likelihood			-125,659			

## C Appendix: Distribution of Expenditure Shares from a Static Problem

To show the implications of the estimated parameter distributions more clearly, following Ameriks, Briggs, Caplin, Shapiro, and Tonetti (2015), I conduct the following exercise. I assume the following maximization problem:

$$\begin{aligned} \text{Max}_{x_1, x_2} & \frac{(x_1 - \kappa)^{1-1/\theta_i}}{1 - 1/\theta_i} + \zeta_{LTC, i} \frac{(x_2 - \kappa_{LTC, i})^{1-1/\theta_i}}{1 - 1/\theta_i} + \zeta_{Beq, i} \frac{(W - x_1 - x_2 - \kappa_{Beq, i})^{1-1/\theta_i}}{1 - 1/\theta_i} \\ \text{s.t. } & 0 \leq x_1, x_2, \leq W \end{aligned} \quad (\text{C.1})$$

which is a static problem with no uncertainty. The household should divide the given resource  $W$  into three expenditures: expenditures in the healthy state ( $x_1$ ), expenditures in the LTC state ( $x_2$ ), and bequests ( $W - x_1 - x_2$ ). Although this problem is unrealistic, the solution can demonstrate the strength of each saving motivation under the estimated parameters. For each individual, I solve (C.1) under  $W = \$400K$  and  $\$1M$  and the proxy values of the preference parameters.<sup>32</sup>

In Figure C1, I show the distribution of expenditure shares. Under  $W = \$400K$ , many respondents spend more than 40 percent of their wealth for expenditure in the LTC state. About 30 percent of respondents do not leave any bequests. Not many respondents spend a large fraction of their resources in the healthy state. When they have more resources ( $W = \$1M$ ), the share of expenditure in the LTC state tends to go down while the share of bequest tends to go up. These changes are driven by the differences in the necessity parameters, which make, on average, bequests luxury goods, and expenditures in the LTC state necessary goods.

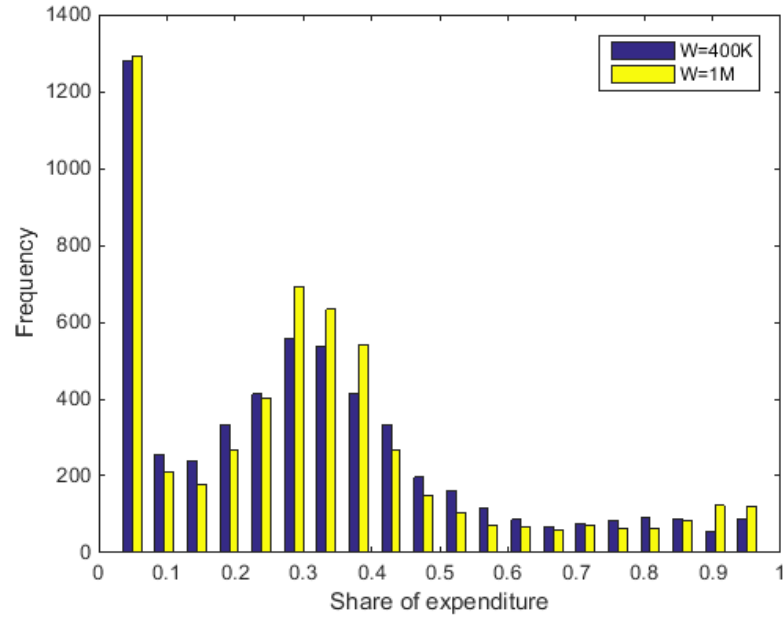
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<sup>32</sup>For the parameters that are assumed to have log-normal distribution, I take the expectation of log of the parameters and take the exponential of it, to avoid Jensen's inequality. The proxy calculated in this way matches the median of the estimated distribution, though it misses the mean, in terms of the level of the parameter. Furthermore, since the proxies are calculated as conditional expectations, it has mean reversion compared to the estimated distribution. Hence plugging these proxies directly into (C.1) would yield less heterogeneity in saving motives compared to what the estimated distributions imply. To correct this problem, in the case of log-normally distributed parameters, before I take exponential of the conditional expectation of the log of the parameter, I convert it by  $\log \Xi' = \log \Xi + (\log \Xi - \mu_\Theta) \frac{\sigma_\Theta}{\sigma_\Xi}$  where  $\log \Xi$  is the expected value of the log of the parameter,  $\mu_\Theta$  is the estimated mean of the log-normal distribution,  $\sigma_\Xi$  is the standard deviation of  $\log \Xi$  and  $\sigma_\Theta$  is the estimated standard deviation of the log-normal distribution. Then I take the exponential of  $\log \Xi'$  to calculate cardinal proxies for this exercise. For normally distributed parameters, I apply the same procedure except for that log operators are dropped in the above formula and do not take exponential at the end.

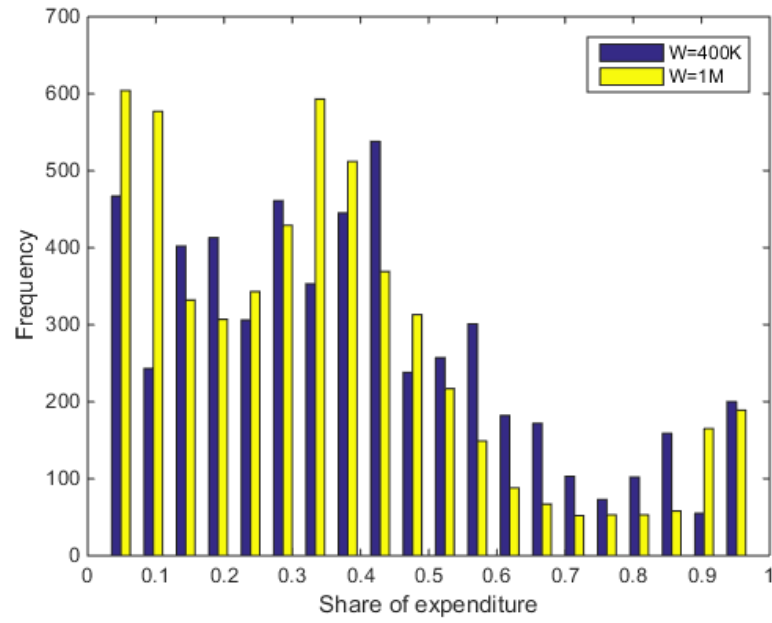


Figure C1: Distribution of Expenditure Share in the Static Problem

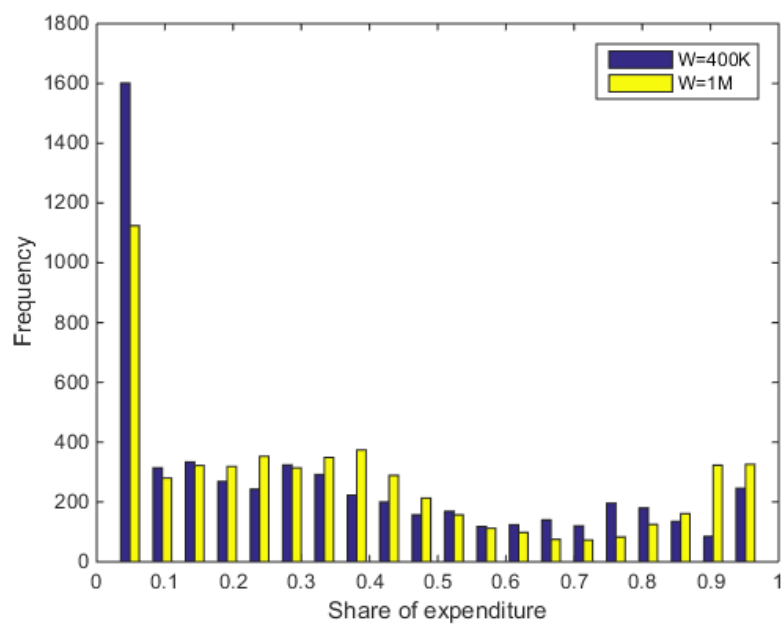
(a) Share of expenditure in the healthy state



(b) Share of expenditure in the LTC state



(c) Share of bequest



Note: N=5,471

## D Appendix: Properties of the Cardinal Proxies Constructed Based on the KSS Approach

In this appendix I explain how using cardinal proxies, constructed under the KSS approach, in place of the true preference parameters can yield unbiased estimates in a linear regression.<sup>33</sup> I also compare the KSS method used in this paper and the individual-level estimation used in Ameriks, Briggs, Caplin, Shapiro, and Tonetti (2016).

Let  $\theta$  denote the true preference parameter. When  $\theta$  is observable, we can run the following regression:

$$y = \beta\theta + \varepsilon \tag{D.1}$$

to estimate the relationship between dependent variable of interest ( $y$ ) and  $\theta$ . In this paper, I cannot observe  $\theta$ , so instead I use the proxy  $\Xi = E[\theta|R]$  where  $R$  represents responses to the survey questions. Let  $u \equiv \theta - \Xi$ . Then the actual regression I run becomes:

$$y = \beta\Xi + \nu \tag{D.2}$$

where  $\nu = \beta u + \varepsilon$ . If  $u$  is a classical measurement error such that it is correlated with  $h$ , then  $h$  is positively correlated with  $\nu$  so the regression yields an attenuation bias. One important virtue of the KSS approach—estimating the population distribution of the parameters first and then calculating cardinal proxies as conditional expectations—is that the resulting cardinal proxy  $\Xi$  is uncorrelated with  $u$  by construction because  $\Xi$  is the result of projection and  $u$  is the error term in that projection. Instead,  $u$  is correlated with the true preference parameter  $\theta$ .

To show this property visually, I run the following simulation. I assume that the true preference parameter is distributed as  $\log(\theta_i) \sim N(\mu, \sigma^2)$ .<sup>34</sup> I do not observe true preference parameters, but I observe survey responses generated as  $R_i = \theta_i + \varepsilon_i$ ,  $\varepsilon_i \sim N(0, \sigma_\varepsilon^2)$ . I assume that  $\mu$  and  $\sigma^2$  are unknown while  $\sigma_\varepsilon^2$  is known.<sup>35</sup> I estimate the unknown parameters using the KSS method and then construct the cardinal proxy  $\Xi$  for  $\theta$ . Figure D1(a) shows the scatter plot of  $\Xi$  and  $\theta$ . Given any level of  $\Xi$ , distribution of observations is symmetric across the 45-degree line, showing that there is no correlation between  $\Xi$  and  $u$ . It is also clear that  $u$  is correlated with  $\theta$ : for large values of  $\theta$

<sup>33</sup>This discussion is based on Kimball, Sahm, and Shapiro (2008).

<sup>34</sup>I chose the functional form to mimic what I used for the risk preference parameter in this paper.

<sup>35</sup>The latter assumption is made for simplicity and does not affect the result in this exercise.

the observations are much more likely to be below the 45-degree line, while the opposite is true for small values of  $\theta$ .<sup>36</sup> This is because of mean reversion in  $\Xi$ , which comes from the fact that  $\Xi$  is calculated as a conditional expectation. A downside of the KSS method is that, due to this mean reversion, the proxy values should not be directly used to calibrate parameters at the individual level in a heterogeneous household model. The degree of heterogeneity in  $\Xi$  is much smaller than the estimated heterogeneity in the population distribution.

An alternative approach is the individual-level estimation used in Ameriks, Briggs, Caplin, Shapiro, and Tonetti (2016). The main difference is that, instead of estimating the population distribution of the parameters, it directly estimates preference parameters at the individual level, using the responses of each individual only. The likelihood function is maximized over individual preference parameters, not over moments of population distribution of the parameters. The parameters estimated in this way can also be considered as the cardinal proxies ( $\Xi$ ) for the parameters for each individual. One advantage of this approach is that it does not make any functional form assumption for the population distribution. Also, there is no mean reversion in  $\Xi$  since  $\Xi$  is not calculated as conditional expectation anymore. The absence of mean reversion makes the estimates attractive for calibrating a heterogeneous household model as in Ameriks, Briggs, Caplin, Shapiro, and Tonetti (2016). One main disadvantage is that now  $\Xi$  is positively correlated with  $u$ , since survey response error directly affects both the point estimate (and hence the proxy) for each individual and the difference between the proxy and the true parameter. The correlation between  $\Xi$  and  $u$  makes the estimates improper as regressors in a linear regression. Figure D1(b) shows the scatter plot between  $\Xi$  and  $u$  under the individual-level estimation, in the same exercise as in D1(a). (In D1(b),  $\Xi$  is nothing but  $R$ .) Under this method,  $u$  is uncorrelated with  $\theta$ : given any level of  $\theta$ , the distribution of observations is symmetrical with respect to the 45-degree line. In contrast, this distribution is correlated with  $\Xi$ : for a large value of  $h$  the observations are more likely to be above the 45-degree line and the opposite is true for a small value of  $\Xi$ .

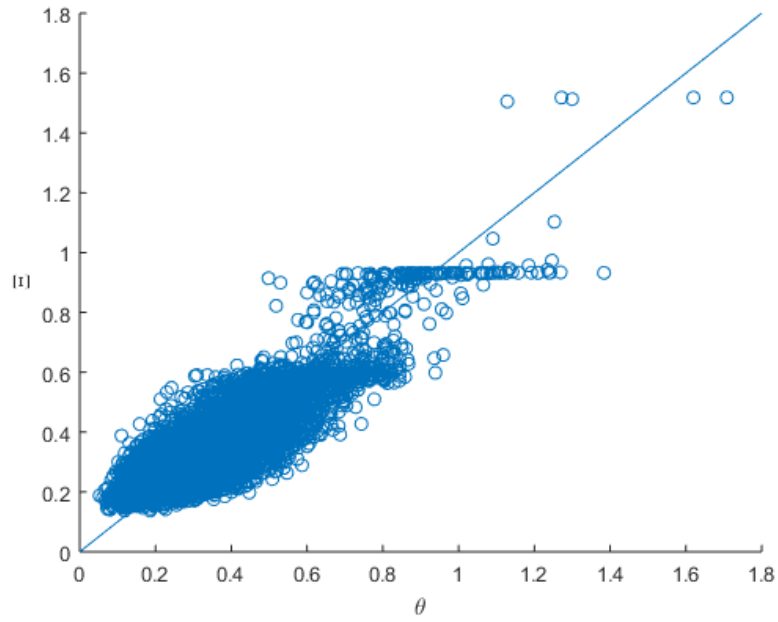
This discussion shows that the choice of the estimation method should be based on how estimates will be used in the analysis.

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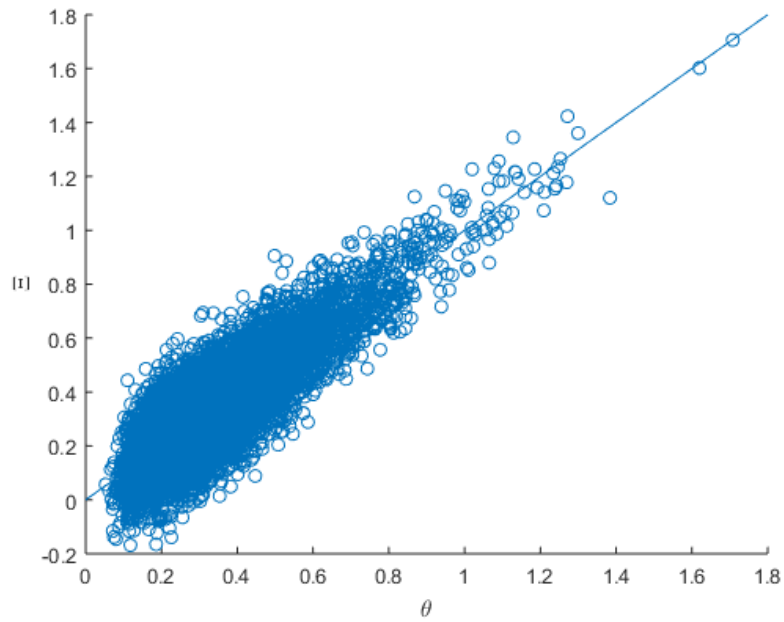
<sup>36</sup>Note that in a linear regression that also has other covariates in the RHS,  $u$  can be correlated with those controls through  $\theta$ , resulting in biased estimates. To correct this, I make the mean of the parameter distributions ( $\mu$ ) a linear function of the controls that will be included in the second stage regression. This would enable us to obtain unbiased estimates in that  $u$  would not be correlated with any of the controls or with the proxy.

Figure D1: Scatter plot of true parameter ( $\theta$ ) and cardinal proxy ( $h$ )

(a) KSS method



(b) Individual-level estimation



## E Appendix: Estimation of Health State Transition Matrix Using the HRS

I use an approach similar to that in De Nardi, French, and Jones (2013). To define health status, I use the self-reported subjective health data from the HRS:  $s = G$  corresponds to  $\{Excellent, Very Good, Good\}$  in the subjective health report and  $s = B$  to  $\{Fair, Poor\}$ . As long as a respondent reports that she requires help for at least one activity of daily living (ADL), then she is classified as  $s = LTC$ .<sup>37</sup> The transition to death ( $s = D$ ) is identified with the exit report in the HRS. I use all the observations from the HRS 2002-2014 for respondents who satisfy the VRI sampling criteria for at least one wave.

Let  $x$  be a vector that includes a constant, age, gender, and square of age and interactions of these variables, as well as indicators for previous health status and previous health interacted with age. I estimate a multinomial logit model, such that for  $i = \{G, B, LTC\}$  and  $j = \{G, B, LTC, D\}$ ,

$$\begin{aligned}\pi_{ij} &= Pr(s' = j | s = i) \\ &= \theta_{ij} / \sum_{k \in \{G, B, LTC, D\}} \theta_{ik} \\ \theta_{iD} &\equiv 1, \forall i \\ \theta_{ik} &\equiv \exp(x\beta_k), \forall i, k \in \{G, B, LTC\}\end{aligned}\tag{E.1}$$

where  $\{\beta_k\}_{k \in \{G, B, LTC\}}$  are sets of coefficient vectors and of course  $Pr(s' = D | s = D) = 1$ .

Note that what I need is an annual transition matrix while the HRS data have information on biennial transitions. These two transition processes are linked, however, by:

$$\begin{aligned}Pr(s'' = j | s = i) &= \sum_k Pr(s'' = j | s' = k) Pr(s' = k | s = i) \\ &= \sum_k \pi_{kj, t+1} \pi_{ik, t}\end{aligned}\tag{E.2}$$

where the subscript  $t$  shows that the transition probability is a function of age (which is part of  $x$ ). (E.1) and (E.2) allow us to estimate  $\{\beta_k\}_{k \in \{G, B, LTC\}}$  directly from the data using maximum likelihood estimation. The transition matrix is built on the estimated coefficients.

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<sup>37</sup>This definition of the LTC state is the closest to that in the VRI survey.